

13.1 Electrostatic superposition, flux and conductors

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS solid angle • induced surface charge

Objective

Add Coulomb forces as vectors using the actual geometry; Compute flux from symmetry or solid angle with a stated orientation; Find conductor fields and potentials using induced surface charges. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Point-charge fields and Coulomb forces add as vectors. Same-direction components add, opposite components subtract, and perpendicular components combine by Pythagoras. Potential instead adds as a scalar, $V = \sum Q/(4\pi\epsilon_0 r)$, with a stated reference. Zero potential does not generally imply zero field: $E = -\nabla V$.

Gauss's law $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\epsilon_0$ concerns a closed surface. Symmetry is needed to infer a simple local field. An infinite open plane subtends solid angle 2π , giving half a point charge's total flux in magnitude. Its sign depends on the normal; the plane does not enclose the charge.

For centred charge Q inside a conducting spherical shell of net charge q , equilibrium zero field in the material induces inner charge $-Q$ and outer charge $Q+q$. With outer radius b and zero potential at infinity, the material potential is $(Q+q)/(4\pi\epsilon_0 b)$, independent of observation radius.

In the cavity, a centred charge gives radial field $Q/(4\pi\epsilon_0 r^2)$. Potential includes constant shell contributions and is continuous across each surface; the normal field can jump. Off-centre Q still induces total inner charge $-Q$, but the cavity field is no longer the centred radial expression.

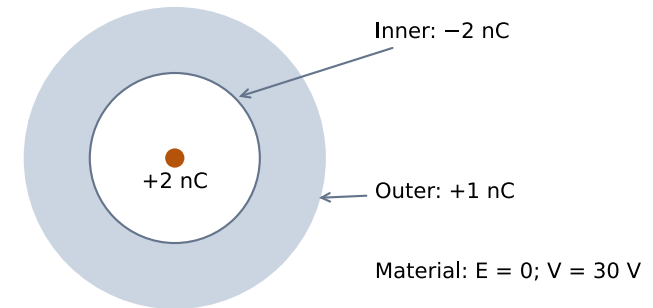
■ Worked method

A centred point charge Q lies in a conducting spherical shell of net charge q . Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{\text{inner}} = -Q$. Charge conservation gives $Q_{\text{outer}} = q+Q$. With outer radius b and zero potential at infinity,

$$V_{\text{metal}} = \frac{q+Q}{4\pi\epsilon_0 b} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(1.0 \times 10^{-9} \text{ C})}{0.30 \text{ m}} = 30 \text{ V}.$$

Here $Q = +2 \text{ nC}$ and $q = -1 \text{ nC}$. The cavity is not conducting material; its field need not vanish.

Shell net charge -1 nC ; outer radius 0.30 m



2 Practice

2.1 A centred charge $+3Q$ lies inside a shell whose net charge is $-Q$. Its outer-surface charge is: [1]

- A. $-3Q$
- B. $-Q$
- C. 0
- D. $+2Q$
- E. $+3Q$

2.2 Two forces of magnitudes 6 N and 8 N are perpendicular. Their resultant magnitude is: [1]

- A. 2 N
- B. 7 N
- C. 10 N
- D. 14 N
- E. 48 N

2.3 The magnitude of flux from a positive point charge through an infinite plane equals _____ times Q/ϵ_0 (decimal). Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Inside equilibrium conducting material, nonzero constant potential is compatible with zero electric field. [2]

3 Exam-style questions

3.1 Move Q away from the centre while keeping it inside the closed cavity. The shell's exterior remains a sphere, with no external charges. Which of inner charge total, inner charge distribution, metal potential and exterior field retain their centred values? Explain using Gauss's law and electrostatic uniqueness. [5]