

46.1 Reaction energies, activity and decay balances

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS Q value Q • activity

Objective

Compute reaction energy from complete atomic or nuclear mass bookkeeping; Calculate activity from half-life and a simple decay-chain balance; Compare fission and fusion binding changes with charge and nucleon conservation. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

$Q = (m_{\text{initial}} - m_{\text{final}})c^2$ with $1 \text{ u} = 931.5 \text{ MeV}$. Atomic masses balance electrons directly for β^- and neutral-He α ; β^+ needs explicit electron masses.

$A = \lambda N$ with $\lambda = \ln 2 / T_{1/2}$. Equal activities need not mean equal atom counts.

Chains: $dN_B/dt = \lambda_A N_A - \lambda_B N_B$; a long-lived parent gives secular equilibrium of activities.

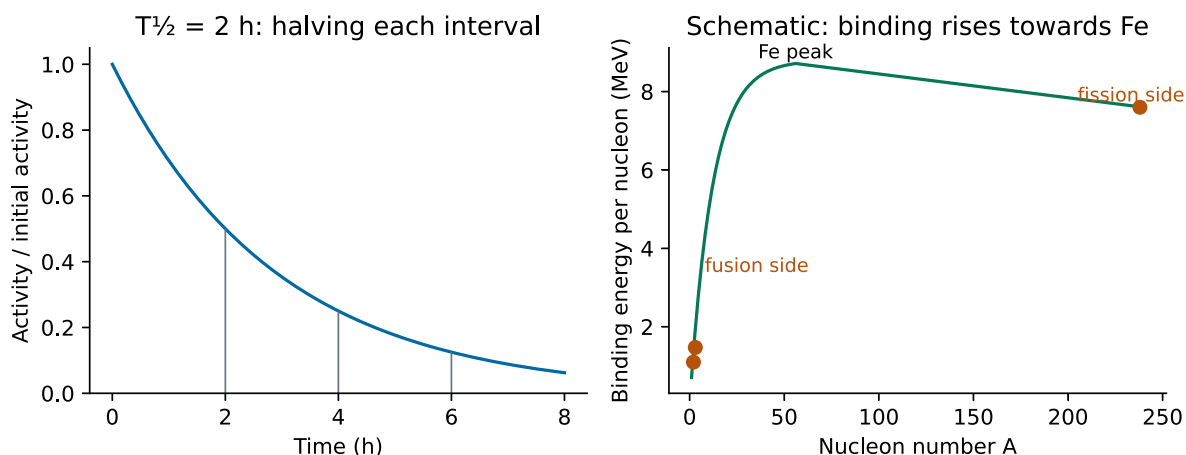
The binding-per-nucleon curve peaks near iron: heavy fission and light fusion both raise total binding and release energy.

■ Worked method

Activity is $A = \lambda N$, with $\lambda = \ln 2 / T_{1/2}$. For $N = 6.0 \times 10^{12}$ and half-life 2 hours, convert time before calculating.

$$A = (\ln 2)N/T_{1/2} = (\ln 2)(6.0 \times 10^{12})/(7200 \text{ s}) = 5.78 \times 10^8 \text{ Bq}.$$

A decay chain requires both production and loss terms: $\dot{N}_B = \lambda_A N_A - \lambda_B N_B$. Equal activities do not imply equal populations. Atomic-mass beta-plus Q values require subtracting $2m_e c^2$.



2 Practice

2.1 Choose one answer. Using atomic masses 238.0508 u (^{238}U), 234.0436 u (^{234}Th) and 4.0026 u (^4He), the alpha-decay Q value Q is about: [1]

- **A** 0.46 MeV
- **B** 2.14 MeV
- **C** 4.28 MeV
- **D** 6.55 MeV
- **E** 9.31 MeV

2.2 Choose one answer. Two samples contain equal numbers of radioactive nuclei; sample X has half the half-life of sample Y. The activity of X is: [1]

- **A** half that of Y
- **B** equal to that of Y
- **C** twice that of Y
- **D** four times that of Y
- **E** one quarter that of Y

2.3 A sample starts with activity 8×10^8 Bq and half-life 3 h. After 9 h its activity is _____ $\times 10^8$ Bq. Show your relation and units. [2]

Solutions

2.1 C $\Delta m = 0.0046 \text{ u}$; $0.0046 \times 931.5 \approx 4.28 \text{ MeV}$.

Compare the rejected options: 0.46 MeV; 2.14 MeV; 6.55 MeV; 9.31 MeV. Use the stated model and conditions.

2.2 C $A = \lambda N$ and $\lambda = \ln 2 / T_{1/2}$, so halving $T_{1/2}$ doubles A at fixed N .

Compare the rejected options: half that of Y ; equal to that of Y ; four times that of Y ; one quarter that of Y . Use the stated model and conditions.

2.3 1. Nine hours is three half-lives: $8 \times 10^8 / 8 = 1 \times 10^8 \text{ Bq}$.

2.4 False. Atomic electron counts already balance for β^- (and for α with neutral He); β^+ needs the explicit $2m_e$.

3.1 Parent population is $N_A = N_0 e^{-\lambda t}$. Daughter equation $\dot{N}_B + \lambda N_B = \lambda N_0 e^{-\lambda t}$ has integrating factor $e^{\lambda t}$, giving $N_B = \lambda N_0 t e^{-\lambda t}$. Derivative is $\lambda N_0 e^{-\lambda t} (1 - \lambda t)$; maximum occurs at $t = 1/\lambda$, with $N_B = N_0/e$. Award parent, equation, integrating factor/solution, maximum time and value. Direct use of an unequal-decay-constant formula would give a removable zero-over-zero limit.