

28.1 Fluid continuity, pressure energy and viscous flow

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS volume flow rate • dynamic viscosity

Objective

Combine incompressible continuity with pressure, height and kinetic energy; Apply hydrostatic and Bernoulli relations only under their stated conditions; Use viscosity, Poiseuille scaling and Reynolds number to distinguish flow regimes. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Steady mass continuity gives $\rho Av = \text{constant}$; incompressible flow gives $Q = Av$. Halving radius quarters area and quadruples speed at fixed Q . Hydrostatic pressure obeys $dp/dz = -\rho g$ with z upward.

Bernoulli $p + \rho v^2/2 + \rho gz = \text{constant}$ applies along a steady incompressible inviscid streamline without pumps/losses. At equal height, ideal pressure falls when speed rises; a stagnation point converts speed energy to pressure.

For fully developed laminar Newtonian flow in a circular tube, $Q = \pi R^4 \Delta p / (8\eta L)$. Doubling R multiplies Q by sixteen at fixed Δp . Viscous pressure loss requires a loss term, not unchanged Bernoulli.

$Re = \rho v D / \eta$ compares inertial and viscous effects and is dimensionless. For a pipe use mean speed and diameter. Transition depends on geometry/disturbances; check regime and fixed quantities before choosing scaling.

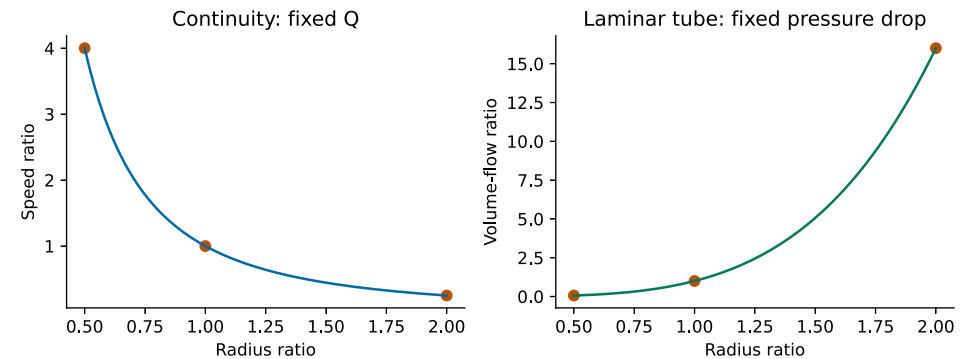
■ Worked method

A steady incompressible horizontal ideal flow has areas 4 and 1 square centimetres. At the first section speed is 1 m/s. Continuity and Bernoulli give

$$v_2 = A_1 v_1 / A_2 = (4 \text{ cm}^2)(1 \text{ m/s}) / (1 \text{ cm}^2) = 4 \text{ m/s}.$$

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) = \frac{1}{2} (1000 \text{ kg/m}^3) [(4 \text{ m/s})^2 - (1 \text{ m/s})^2] = 7500 \text{ Pa}.$$

These relations assume no pump, viscosity loss or height change. A real narrow tube may violate them.



2 Practice

2.1 Choose one answer. At fixed steady incompressible Q , pipe radius is halved. Mean speed becomes: [1]

- A one quarter
- B one half
- C unchanged
- D twice
- E four times

2.2 Choose one answer. For fully developed laminar tube flow, radius doubles while Δp , η and L stay fixed. Q becomes: [1]

- A twice
- B four times
- C eight times
- D sixteen times
- E unchanged

2.3 A stationary liquid with $\rho = 800 \text{ kg/m}^3$ and $g = 10$ has a gauge-pressure rise of _____ Pa at depth 0.5 m. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: Loss-free Bernoulli can be used unchanged to describe a finite viscous pressure drop in a uniform horizontal tube. [2]

3 Exam-style questions

3.1 Two fully developed laminar Newtonian tubes of equal length and viscosity connect in series. Their radii are R and $2R$. Find their pressure-drop ratio and equivalent hydraulic resistance. Compare with connecting the same tubes in parallel. [5]

Solutions

2.1 E Area is proportional to radius squared, so the area quarters and speed quadruples.

Compare the rejected options: one quarter; one half; unchanged; twice. Use the stated model and conditions.

2.2 D Poiseuille flow has Q proportional to R^4 under these stated conditions.

Compare the rejected options: twice; four times; eight times; unchanged. Use the stated model and conditions.

2.3 4000. $\rho gh = 800 \cdot 10 \cdot 0.5 = 4000$.

2.4 False. Equal speed and height would predict equal pressure; a viscous loss requires an additional term/model.

3.1 Define resistance $\mathcal{R} = \Delta p / Q = 8\eta L / (\pi R^4)$ for the narrow tube. Wide-tube resistance is $\mathcal{R}/16$. In series Q is common, pressure ratio is 16:1, and total resistance $17\mathcal{R}/16$. In parallel pressure is common, flow ratio narrow:wide is 1:16 and equivalent resistance $\mathcal{R}/17$. Award radius scaling, series ratio, series total, parallel ratio and parallel total.