

27.1 Accelerating and rotating reference frames

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS Coriolis force • centrifugal force

Objective

Transform acceleration with translating and rotating frame terms; Determine centrifugal, Coriolis and Euler directions from cross products; Separate apparent forces from real interactions and test inertial limits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

A translating origin with acceleration A adds $-mA$ to the frame force equation: $m a' = F_{\text{real}} - mA$. For an upward-accelerating elevator, $N = m(g+a)$. A uniformly translating frame adds no apparent force.

Rotating axes give $a = a_{\text{origin}} + a' + 2\Omega \times v' + \Omega \times (\Omega \times r) + \dot{\Omega} \times r$. Moving the extra terms to the force side gives Coriolis, centrifugal and Euler apparent forces. Use relative v' and one consistent vector basis.

For Ω along $+z$, r and outward v' along $+x$, centrifugal force points $+x$ while Coriolis points $-y$. Reversing v' reverses Coriolis. F_C is perpendicular to v' and does no instantaneous relative work.

A body fixed on a constant-speed platform has $v' = a' = 0$; its real inward force cancels the outward centrifugal term. Constant Ω removes Euler only. The $\Omega \rightarrow 0$ and $A \rightarrow 0$ limits recover ordinary Newtonian motion.

■ Worked method

In a rotating frame, Coriolis force uses the relative velocity.

$$\mathbf{F}_C = -2m\boldsymbol{\Omega} \times \mathbf{v}', \quad \mathbf{F}_{cf} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$$

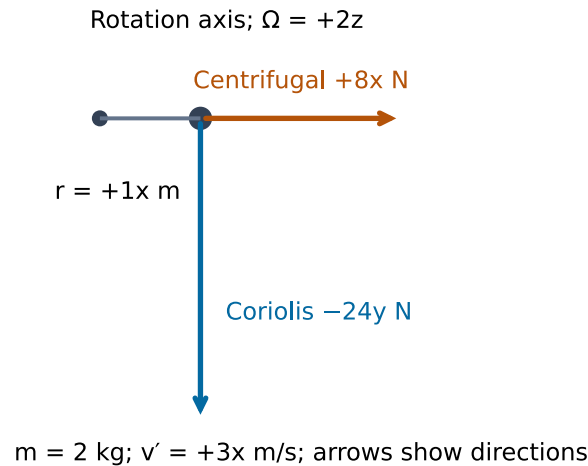
For $m = 2 \text{ kg}$, $\Omega = 2 \text{ rad/s}$ along z , $r = 1 \text{ m}$ along x and $v' = 3 \text{ m/s}$ along x ,

$$F_{cf} = m\Omega^2 r = (2 \text{ kg})(2 \text{ rad/s})^2(1 \text{ m}) = 8 \text{ N} \quad (+x).$$

$$F_C = 2m\Omega v' = 2(2 \text{ kg})(2 \text{ rad/s})(3 \text{ m/s}) = 24 \text{ N} \quad (-y).$$

Euler force is zero only because Ω is constant. These are coordinate terms, not extra physical contacts.

Constant anticlockwise frame; apparent forces



2 Practice

2.1 Choose one answer. Ω points $+\hat{z}$ and v' points $+\hat{x}$. The Coriolis force points: [1]

- **A** $+\hat{x}$
- **B** $-\hat{x}$
- **C** $+\hat{y}$
- **D** $-\hat{y}$
- **E** $+\hat{z}$

2.2 Choose one answer. A mass stays fixed at nonzero radius on a platform with constant rotation. Which statement is correct? [1]

- **A** Its inertial acceleration is zero
- **B** Its Coriolis force is nonzero because the platform turns
- **C** A real inward force balances the rotating-frame centrifugal term
- **D** Centrifugal force is an additional real interaction
- **E** Its real force must be outward

2.3 A 3 kg passenger accelerates downward at 2 m/s^2 with $g=10$. The scale force is _____ N. Show your relation and units. [2]

Solutions

2.1 D $z \times x = y$; the minus sign in $-2m\Omega \times v'$ reverses it.

Compare the rejected options: $+x$; $-x$; $+y$; $+z$. Use the stated model and conditions.

2.2 C $v' = 0$ removes Coriolis; the real centripetal force is inward even though $a' = 0$.

Compare the rejected options: Its inertial acceleration is zero; Its Coriolis force is nonzero because the platform turns; Centrifugal force is an additional real interaction; Its real force must be outward. Use the stated model and conditions.

2.3 24. $N - 30 = 3(-2)$, so $N = 24$.

2.4 True. The cross product is perpendicular to v' , so $F_C \cdot v' = 0$.

3.1 With $v' = a' = 0$, the inertial acceleration is $\Omega \times (\Omega \times r) + \dot{\Omega} \times r = -\Omega^2 r \hat{x} + \alpha r \hat{y}$. Real force is $m(-\Omega^2 r \hat{x} + \alpha r \hat{y})$. Coriolis vanishes, while the Euler apparent force points $-y$ and centrifugal points $+x$. Award centripetal term, tangential term, real force and apparent-force distinction.