

24.1 Scattering, degeneracy and Pauli operators

Name: _____ Class: _____ Date: _____

Total: 11 marks**KEY WORDS** reflection coefficient • degeneracy

Objective

Match travelling waves and fluxes across finite potential changes; Count degenerate isotropic-oscillator states; Evaluate Pauli products and distinguish anticommutation from equality. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For constant potential with $E > V$, $k = \sqrt{2m(E-V)}/\hbar$ and solutions travel as $\exp(\pm ikx)$. At finite steps of equal mass, ψ and ψ' are continuous. No incoming wave from the right leaves only a right-travelling transmitted wave. A finite well returning to its external potential has equal incident/transmitted external k .

Step matching gives $r = (k_1 - k_2)/(k_1 + k_2)$, $t = 2k_1/(k_1 + k_2)$. Reflection $R = |r|^2$; transmission $T = (k_2/k_1)|t|^2$ uses probability current. $R + T = 1$ for a lossless step. Downward steps can reflect; raw squared transmitted amplitude is not always its current fraction.

A spin-zero 3D isotropic oscillator has energy $(N+3/2)\hbar\omega$, $N = n_x + n_y + n_z \geq 0$. Ordered nonnegative triples number $(N+1)(N+2)/2$. For $N=2$, permutations of $(2,0,0)$ and $(1,1,0)$ give six states. Extra spin/identical-particle constraints require separate counting.

Pauli matrices $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ each square to I . Cyclic products give $\sigma_x \sigma_y = i\sigma_z$; reversed order changes sign. Thus distinct matrices anticommute, and $\sigma_x \sigma_z = -i\sigma_y$. Physical spin is $S_i = \hbar \sigma_i / 2$.

■ Worked method

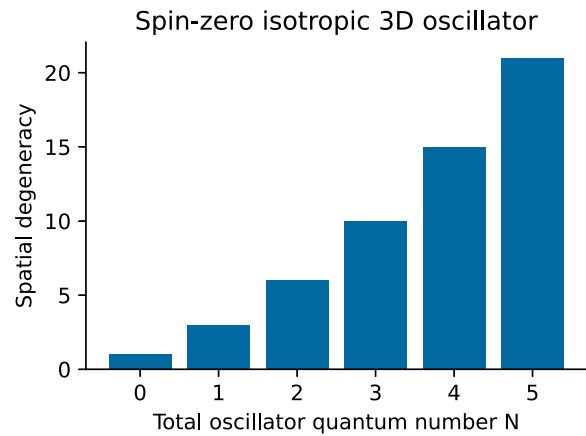
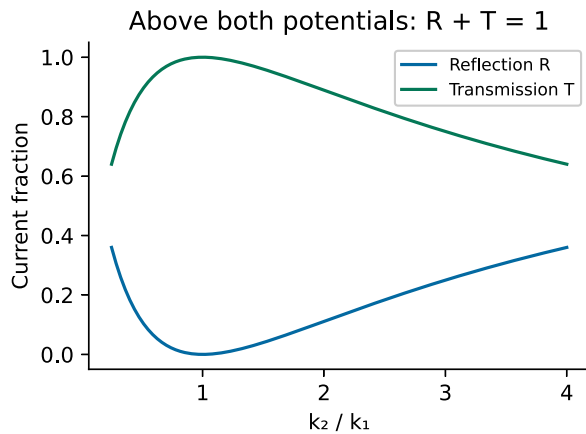
At a finite potential step, match wavefunction and derivative for the same particle mass.

$$r = \frac{k_1 - k_2}{k_1 + k_2}, \quad t = \frac{2k_1}{k_1 + k_2}.$$

Transmission probability uses current, not amplitude alone. For $k_2 = 3k_1$, $r = -1/2$, $t = 1/2$.

$$R = |r|^2 = 1/4, \quad T = (k_2/k_1)|t|^2 = 3/4, \quad R + T = 1.$$

Reflection occurs even for a downward step. Its phase sign disappears when taking probability.



2 Practice

2.1 Choose one answer. For a lossless potential step with $k_2=2k_1$, reflection probability is: [1]

- A 0
- B $1/9$
- C $1/3$
- D $4/9$
- E 1

2.2 Choose one answer. A spin-zero three-dimensional isotropic oscillator at $E=(5/2)\hbar\omega$ has degeneracy: [1]

- A 1
- B 2
- C 3
- D 4
- E 6

2.3 For a spin-zero isotropic oscillator with $N=4$, spatial degeneracy is _____. Show your relation and units. [2]

Solutions

2.1 B $r=(1-2)/(1+2)=-1/3$; $R=1/9$.

Compare the rejected options: 0; $1/3$; $4/9$; 1. Use the stated model and conditions.

2.2 C $N=1$ gives triples $(1,0,0), (0,1,0), (0,0,1)$.

Compare the rejected options: 1; 2; 4; 6. Use the stated model and conditions.

2.3 15. $(N+1)(N+2)/2=5 \cdot 6/2=15$.

2.4 False. Their products change sign on reversing order; they anticommute.

3.1 Norm is $(3+1)/4 = 1$. S_z gives $+\hbar/2$ with probability $3/4$ and $-\hbar/2$ with probability $1/4$. The x eigenstates are $(1, \pm 1)^T/\sqrt{2}$. Their amplitudes are $(\sqrt{3} \pm i)/(2\sqrt{2})$, each with squared modulus $1/2$. Award norm, z results/probabilities, x eigenbasis, projections and x probabilities. Complex conjugation is required in the inner product; the imaginary relative phase matters.