

12.1 Coupled acceleration and relative motion

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS constraint • relative velocity

Objective

Solve coupled-body equations using one acceleration constraint ; Transform velocities between inertial frames; Check force and energy limits for constrained motion. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For table mass m_1 and hanging mass m_2 connected over an ideal pulley, a taut inextensible string gives one acceleration magnitude. Write $T=m_1a$ and $m_2g-T=m_2a$. Thus $a=m_2g/(m_1+m_2)$, $T=m_1m_2g/(m_1+m_2)<m_2g$. Equating tension to weight incorrectly assumes no acceleration.

Equal tensions require an ideal massless string/pulley without friction or inertia. For pulley inertia I , use $(T_2-T_1)R=I\alpha$ and $a=R\alpha$. Draw separate bodies: a force internal to the combined system remains external to an individual body.

Galilean velocity transformation is $v_{\text{relative}}=v_{\text{ground}}-V$. A load released from a constant-speed level aircraft retains its horizontal ground velocity. With upward positive and no drag, vertical velocity is $-gt$. Relative horizontal velocity is zero; vertical displacement is $-\frac{1}{2}gt^2$, which is a different quantity.

Check limits: $m_1\rightarrow 0$ gives $a\rightarrow g$ and $T\rightarrow 0$; large m_1 gives $a\rightarrow 0$ and $T\rightarrow m_2g$ from below. Release-from-rest energy gives $m_2gs=\frac{1}{2}(m_1+m_2)v^2$, agreeing with $v^2=2as$. This check assumes no friction or other unaccounted work.

■ Worked method

A table mass m_1 and hanging mass m_2 share an acceleration through a taut ideal string. Choose rightward table motion and downward hanging motion positive.

$$\begin{aligned}T &= m_1a, & m_2g - T &= m_2a. \\a &= \frac{m_2g}{m_1 + m_2} = \frac{(2\text{ kg})(10\text{ m/s}^2)}{3\text{ kg} + 2\text{ kg}} = 4\text{ m/s}^2. \\T &= m_1a = (3\text{ kg})(4\text{ m/s}^2) = 12\text{ N}.\end{aligned}$$

T is less than the hanging weight 20 N. A nonzero net downward force is needed for acceleration.

Subtract aircraft velocity; components in m/s

Velocity after 3 s	Horizontal	Vertical
Ground	80	−30
Aircraft	80	0
Payload relative to aircraft	0	−30

2 Practice

2.1 Choose one answer. For $m_1=4\text{ kg}$ on a frictionless table and $m_2=1\text{ kg}$ hanging, $g=10\text{ m/s}^2$, tension is: [1]

- **A** 10 N
- **B** 2 N
- **C** 8 N
- **D** 40 N
- **E** 0 N

2.2 Choose one answer. A payload is dropped from a constant-speed level aircraft. With $g=10\text{ m/s}^2$ and no drag, its velocity relative to that aircraft after 2 s is: [1]

- **A** 20 m/s downward
- **B** 20 m/s forward
- **C** 40 m/s downward
- **D** Zero
- **E** The aircraft speed forward plus 20 m/s downward

2.3 For $m_1=m_2$ and an ideal frictionless table/pulley, $a/g=$ _____ (decimal). Show your relation and units. [2]

[2]

[5]

Solutions

2.1 C $a=10/5=2 \text{ m/s}^2$ and $T=m_1a=8 \text{ N}$.

Compare the rejected options: 10 N; 2 N; 40 N; 0 N. Use the stated model and conditions.

2.2 A Horizontal velocities cancel; vertical relative speed is $gt=20 \text{ m/s}$ downward.

Compare the rejected options: 20 m/s forward; 40 m/s downward; Zero; The aircraft speed forward plus 20 m/s downward. Use the stated model and conditions.

2.3 0.5. $a=m_2g/(m_1+m_2)=g/2$.

2.4 True. Net forces are m_1a and m_2a ; equal acceleration does not imply equal masses or forces.

3.1 $T_1 = m_1a$; $m_2g - T_2 = m_2a$; $(T_2 - T_1)R = I\alpha$. With $I = MR^2/2$ and $\alpha = a/R$, $T_2 - T_1 = Ma/2$. Thus $a = m_2g/(m_1 + m_2 + M/2)$, $T_1 = m_1m_2g/(m_1 + m_2 + M/2)$ and $T_2 = m_2(g - a)$. At $M = 0$ tensions become equal and the ideal result returns. Award body equations, torque/constraint, acceleration, tensions and limit.