

Exercise walk-through

41.1

Thermal spectra and one-electron scaling

GRE · GRE Subject Test · GRE Physics

41.1 Thermal spectra and one-electron scaling

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS spectral radiance • series limit

Objective

Distinguish wavelength spectral peak, integrated radiation and photon energy; Derive Bohr radius and energy scaling under one-electron assumptions; Calculate Coulomb-ion transitions, series limits and ionisation thresholds. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

Planck B_λ is spectral radiance per wavelength; $M_\lambda = \pi B_\lambda$ integrates to $\sigma_{\text{SB}} T^4$. Wien's wavelength peak uses $\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$. Frequency and wavelength peaks differ.

For the stated grey surface, $P_{\text{net}} = \epsilon \sigma_{\text{SB}} A (T^4 - T_{\text{env}}^4)$, in Kelvin. Photon energy

hc/λ refers to a selected wavelength, not every photon in a thermal distribution.

Bohr Coulomb force plus $m_e v r = n \hbar$ gives $r_n = a_0 n^2 / Z$, $v_n = Z \alpha c / n$, $E_n \approx -13.6 Z^2 / n^2$ eV. It is a heavy-nucleus one-electron model; wave orbital L and reduced mass need separate treatment.

Emission energy is $13.6 Z^2 (1/n_f^2 - 1/n_i^2)$ eV. Ionisation from n is $13.6 Z^2 / n^2$. At fixed final n, the infinite initial-n series limit gives the shortest wavelength.

■ **Worked method**

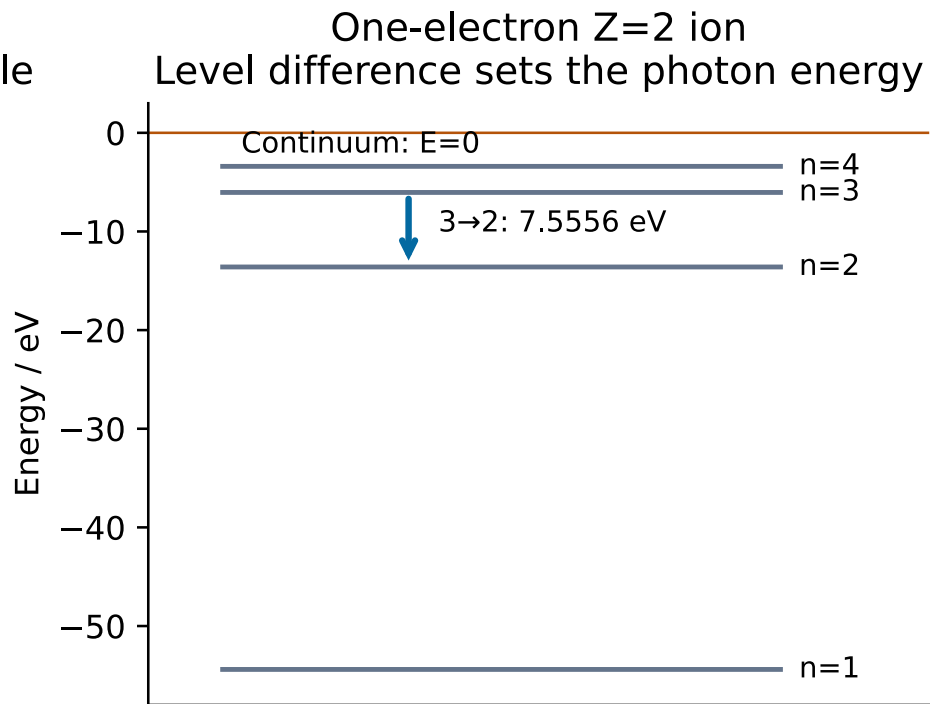
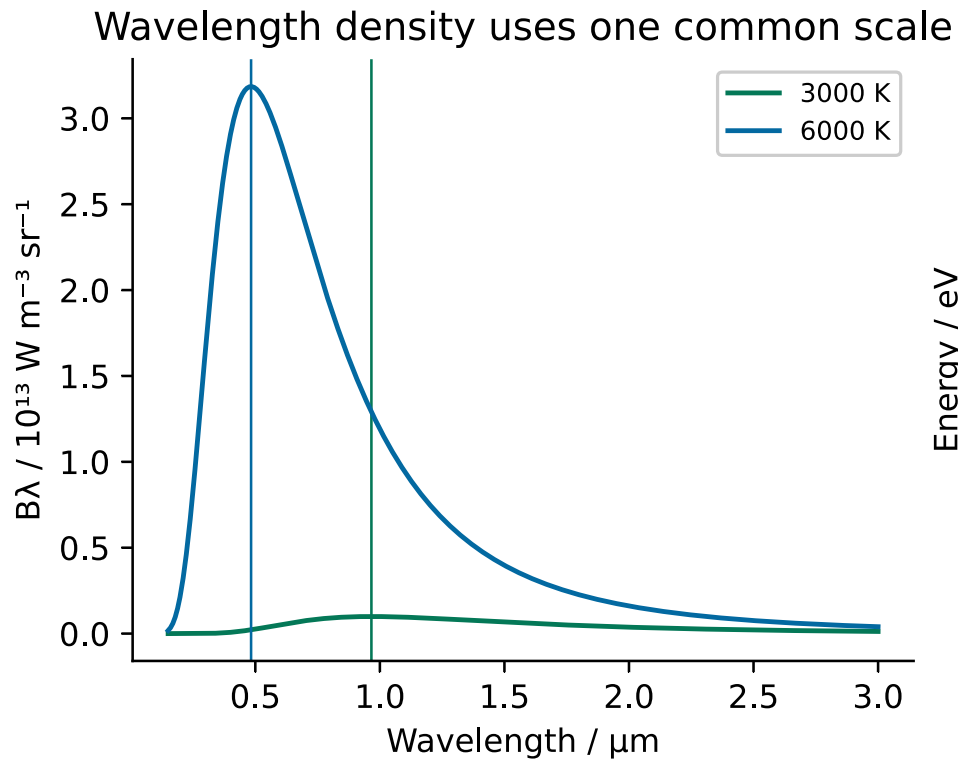
Wien's law refers to the wavelength-density peak of a blackbody spectrum.

$$\lambda_{max} = b/T = (2.898 \times 10^{-3} \text{ m K}) / (3000 \text{ K}) = 966 \text{ nm}.$$

A photon at that wavelength has energy

$$E_{\gamma} = hc/\lambda = (1240 \text{ eV nm})/(966 \text{ nm}) = 1.28 \text{ eV}.$$

The source emits a distribution of photon energies. Its integrated surface flux is σT^4 , and the frequency-density peak is not obtained by simply using c divided by this peak wavelength.



Exercise 2.1

2.1 Choose one answer. An ideal blackbody temperature doubles. Its wavelength peak and integrated emitted surface flux change by: [1]

- **A** Peak doubles; flux doubles
- **B** Peak halves; flux increases $4\times$
- **C** Peak halves; flux increases $16\times$
- **D** Peak unchanged; flux increases $16\times$
- **E** Peak halves; flux unchanged

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2.1 C Wien gives inverse- T wavelength; Stefan–Boltzmann gives T^4 integrated flux.

Compare the rejected options: Peak doubles; flux doubles; Peak halves; flux increases $4\times$; Peak unchanged; flux increases $16\times$; Peak halves; flux unchanged. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. In the heavy-nucleus Bohr one-electron model, $r_{n=2}$ for $Z=2$ is: [1]

- **A** $a_0/2$
- **B** a_0
- **C** $2a_0$
- **D** $4a_0$
- **E** $8a_0$

2.3 Using $\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$, the wavelength peak at 6000 K is _____ nm.

Exercises 2.2-2.3

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- **A** $a_0/2$
- **B** a_0
- **C** $2a_0$
- **D** $4a_0$
- **E** $8a_0$

2.2 C $r=a_0 n^2/Z=a_0 \cdot 4/2=2a_0$.

Compare the rejected options: $a_0/2$; a_0 ; $4a_0$; $8a_0$. Use the stated model and conditions.

2.3 Using $\lambda_{\text{max}} T=2.898 \times 10^{-3} \text{ m} \cdot \text{K}$, the wavelength peak at 6000 K is _____ nm.

2.3 483. $2.898 \times 10^{-3}/6000 \text{ m}=483 \text{ nm}$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: The wavelength-density Planck peak and frequency-density Planck peak are related simply by $\nu_{\text{peak}} = c/\lambda_{\text{peak}}$. [2]

3.1 A grey sphere doubles its radius while temperature changes from T to $2T$ in an environment at temperature T . Compare its final net power to the original emitted power, assuming fixed emissivity. Also find the peak-wavelength ratio. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: The wavelength-density Planck peak and frequency-density Planck peak are related simply by $\nu_{\text{peak}} = c/\lambda_{\text{peak}}$. [2]

2.4 False. Changing the spectral interval introduces a Jacobian and changes the location of the maximum.

3.1 A grey sphere doubles its radius while temperature changes from T to $2T$ in an environment at temperature T . Compare its final net power to the original emitted power, assuming fixed emissivity. Also find the peak-wavelength ratio. [4]

3.1 Original emitted power is $P_0 = \epsilon\sigma 4\pi R^2 T^4$. Final net power is $P_f = \epsilon\sigma 4\pi (2R)^2 [(2T)^4 - T^4] = 60P_0$. Peak wavelength halves. Original net power in that same environment is zero, so a ratio to original NET power would be undefined. Award reference power, final power, peak and reference distinction.