

Exercise walk-through

30.1

RC and RL transients, impedance and resonance

GRE · GRE Subject Test · GRE Physics

30.1 RC and RL transients, impedance and resonance

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS time constant • impedance

Objective

Solve RC charging and discharging with stated initial conditions; Solve RL current response and account for stored energy; Use complex impedance, phase and resonance in sinusoidal circuits. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For RC, $R \frac{dq}{dt} + q/C = V_s$ and $\tau = RC$. $V_C = V_s + (V_C(0) - V_s) \exp(-t/\tau)$. An uncharged capacitor reaches 63.2% after τ ; discharge leaves 36.8%. Voltage is continuous without impulsive current.

For RL, $L \frac{dI}{dt} + RI = V_s$ and $\tau = L/R$. $I = V_s/R + (I(0) - V_s/R) \exp(-t/\tau)$. Current is continuous for finite voltage. Stored energy $LI^2/2$ becomes resistor heat during decay.

Using $\exp(i\omega t)$, series RLC has $Z = R + iX$ with $X = \omega L - 1/(\omega C)$. $I_{\text{rms}} = V_{\text{rms}}/\sqrt{R^2 + X^2}$. current lags for $X > 0$ and leads for $X < 0$. Add complex impedances, not magnitudes.

Series driven resonance has $\omega_0 = 1/\sqrt{LC}$, $X = 0$ and maximal current. Mean power is $I_{\text{rms}}^2 R$. Damped free oscillation instead has frequency $\sqrt{1/(LC) - (R/(2L))^2}$ when underdamped. Distinguish the drive and regime.

■ Worked method

An RC circuit switches to fixed source voltage V_s . Charge conservation and Kirchhoff's law give

$$RC\dot{V}_C + V_C = V_s, \quad V_C(t) = V_s + [V_C(0) - V_s]e^{-t/(RC)}.$$

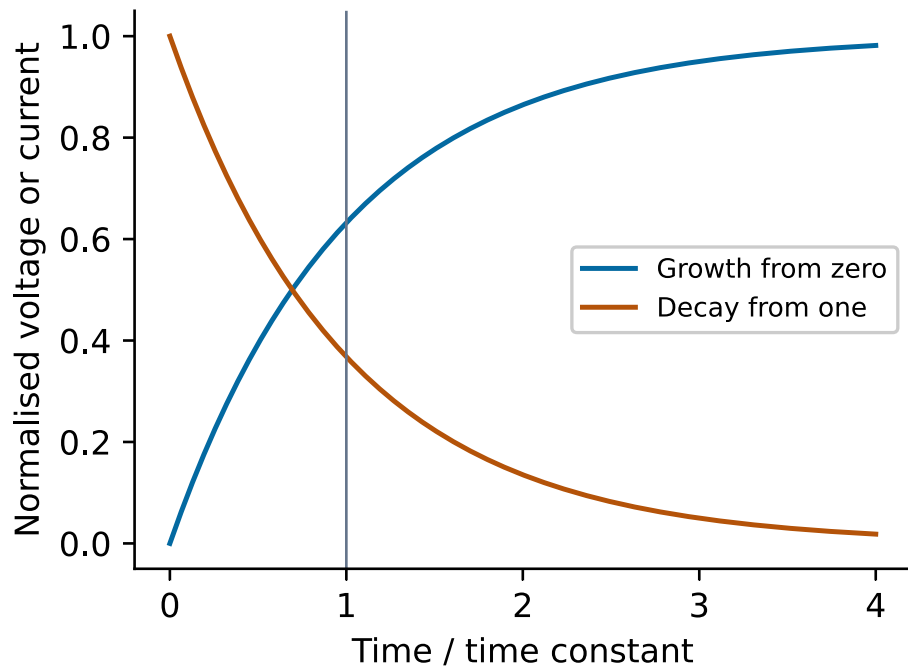
For $R = 2$ kilohms, $C = 1$ millifarad, $V_s = 12$ V and initial voltage zero,

$$\tau = RC = (2000\ \Omega)(0.001\ \text{F}) = 2.0\ \text{s}.$$

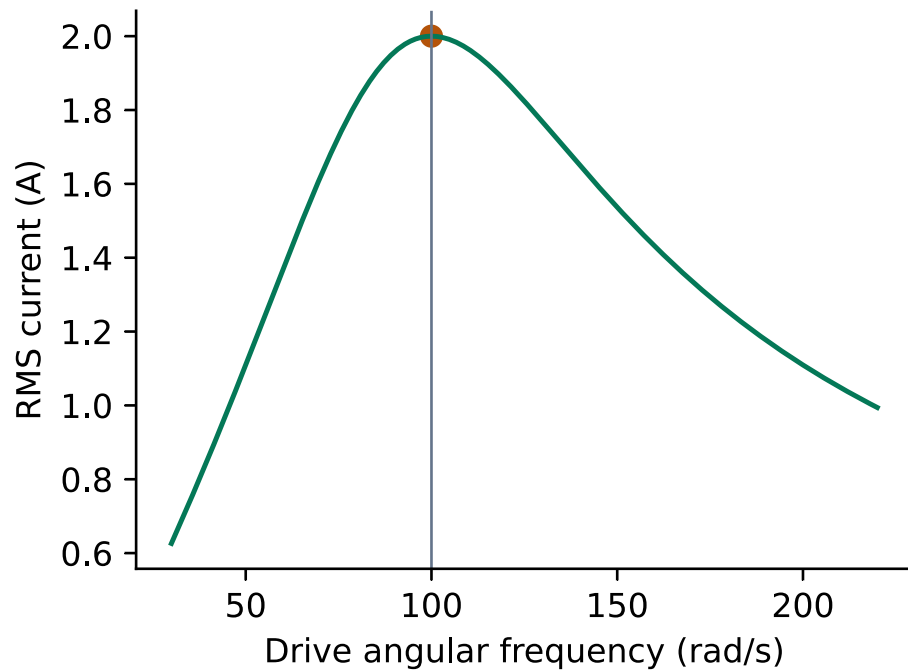
$$V_C(\tau) = V_s(1 - e^{-1}) = (12\ \text{V})(1 - e^{-1}) = 7.59\ \text{V}.$$

The time constant controls the remaining difference from the final state.

RC: $\tau = RC$; RL: $\tau = L/R$



Series RLC: resonance at 100 rad/s



Exercise 2.1

2.1 Choose one answer. After one time constant, an initially uncharged RC capacitor driven by a constant source reaches approximately: [1]

- **A** 36.8% of final voltage
- **B** 50% of final voltage
- **C** 63.2% of final voltage
- **D** 86.5% of final voltage
- **E** 100% of final voltage

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2.1 C $V_C/V_s = 1 - \exp(-1) \approx 0.632$; $\exp(-1)$ is the remaining difference.

Compare the rejected options: 36.8% of final voltage; 50% of final voltage; 86.5% of final voltage; 100% of final voltage. Use the stated model and conditions.

Exercises 2.2·2.3

2.2 Choose one answer. Using $\exp(i\omega t)$, a series circuit with $X = \omega L - 1/(\omega C) > 0$ has: [1]

- **A** Current leading voltage
- **B** Current lagging voltage
- **C** Voltage and current necessarily in phase
- **D** Zero resistor dissipation
- **E** Impedance magnitude $R + X$

2.3 An RL circuit has $L = 0.6$ H and $R = 3$ Ω . Its time constant is _____ s. Show your relation and units. [2]

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2.2 B Z has a positive phase; $I = V/Z$ has a negative phase relative to voltage.

Compare the rejected options: Current leading voltage; Voltage and current necessarily in phase; Zero resistor dissipation; Impedance magnitude $R + X$. Use the stated model and conditions.

2.3 An RL circuit has $L = 0.6$ H and $R = 3$ Ω . Its time constant is _____ s. Show your relation and units. [2]

2.3 0.2. $\tau = L/R = 0.6/3 = 0.2$ s.

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: An ideal inductor stores energy but dissipates no average real power in sinusoidal steady state. [2]

3.1 An RC capacitor starts at 8 V and switches to a -4 V ideal source through R. Find its voltage at arbitrary t , the first zero crossing and the initial current defined into its positive-voltage plate. Explain voltage continuity. [5]

Exercises 2.4·3.1

2.4 Decide whether this claim is true. Justify it: An ideal inductor stores energy but dissipates no average real power in sinusoidal steady state. [2]

2.4 True. Its ideal voltage/current phase is a quarter cycle; average power is zero while $LI^2/2$ varies.

3.1 An RC capacitor starts at 8 V and switches to a -4 V ideal source through R. Find its voltage at arbitrary t, the first zero crossing and the initial current defined into its positive-voltage plate. Explain voltage continuity. [5]

3.1 With $\tau = RC$, $V_C = -4 + 12e^{-t/\tau}$ volts. Zero crossing satisfies $e^{-t/\tau} = 1/3$, so