

Solutions

2.1 C. Cross-section area is $\pi(2Rh - h^2) = 5\pi$. Multiply by depth-decrease magnitude $1/4$. The section area is five pi and the speed magnitude one-quarter. The other coefficients use radius, whole-sphere quantities or omit the correct section factor.

2.2 C. The quotient is $|x|$. Filling in zero gives continuity, while the left and right derivatives are -1 and 1 . Filling zero removes the missing value, so the extension is continuous. Its unequal one-sided slopes rule out differentiability with derivative one; it is not constant.

2.3 At depth s , the cross-sectional radius obeys $r(s)^2 = 2Rs - s^2$. Integrating disks gives $V(h) = \pi(Rh^2 - h^3/3)$ for $0 \leq h \leq 2R$.

$$\frac{dV}{dt} = \pi(2Rh - h^2) \frac{dh}{dt}.$$

$$\frac{dV}{dt} = \pi[2(2 \text{ m})(1 \text{ m}) - (1 \text{ m})^2](-0.1 \text{ m/min}) = -0.3\pi \text{ m}^3/\text{min}.$$

The outflow magnitude is $0.3\pi \text{ m}^3/\text{min}$. The sign describes loss of stored water, not negative physical outflow.

3.1 Since $g'(0)$ is nonzero, the denominator is nonzero near zero except at zero. Factor the expression as $(f/g)(f - 1)/(2 - g^2)$. Its limit is $(4/2)(-1)/2 = -1$. First-order differentiability gives $f/g \rightarrow f'(0)/g'(0)$; it does not give second-order control. For a counterexample preserving these derivatives, take $f(x) = 4x + x|x|$ and $g(x) = 2x$. They are continuously differentiable. For nonzero x the quotient becomes $(2 + |x|/2)(-1 + 4x + x|x|)/(2 - 4x^2)$. Its expansion is $-1 + 4x - |x|/4 + O(x^2)$, giving different one-sided derivatives. Thus the extension is continuous but not differentiable.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.