

Solutions

2.1 B. Integration by parts gives uv minus the remaining integral. The lower boundary is zero for $n \geq 1$. The plus-sign choice violates integration by parts; the version without x has wrong boundary term. Multiplying by x or by n alone omits the required boundary expression.

2.2 B. At each y , x ranges from 0 to y . The inner integral supplies length y . Reversal gives inner length y . Unit length, one minus y and y squared describe different regions; e to y changes the given integrand.

2.3 The region is $0 \leq x \leq y \leq 1$. In the reversed order, y ranges from zero to one and x from zero to y . Continuity on the compact triangle permits order reversal, giving $\int_0^1 ye^{y^2} dy = (e - 1)/2$. Integration by parts with $u = (\ln t)^2$, $dv = dt$ gives $I_2 = x(\ln x)^2 - 2I_1$. Since $I_1 = x \ln x - x + 1$,

$$I_2 = x[(\ln x)^2 - 2 \ln x + 2] - 2.$$

Differentiation checks the integrand; substituting $x=1$ checks the constant.

3.1 The endpoint limits are $2m$ at zero and $-2m$ at π , by the first-order sine limit. Both singularities are removable, so the continuously extended integrand is integrable. Reflection $x \mapsto \pi - x$ preserves the denominator and reverses the numerator. The integral equals its negative and therefore is zero. Symmetric cancellation would not establish convergence for a genuinely divergent endpoint; the removable limits are essential.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.