

Solutions

2.1 D. The inverse matrix of $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ is $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$. Its top-left entry is $u_x=2$. Holding y fixed forces u and v to change together. The top-left inverse entry is two, ruling out zero, one-half, one and minus one; one-half would invert a single partial while fixing the wrong variable.

2.2 A. $\det J=-3$. The bottom-right inverse entry is $2/(-3)=-2/3$. The determinant is minus three and the required cofactor is two. The other choices use a different entry, miss the determinant or reverse its sign.

2.3 The corresponding output is $(1, 1)$. The Jacobian is $J = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$ with determinant -3 , nonzero. Continuous partials give a differentiable local inverse. Thus

$$J^{-1} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{pmatrix}.$$

Read rows as (u_x, u_y) and (v_x, v_y) , all at output $(1, 1)$. Multiplication gives the identity. In particular $u_x = 1/3$, not $1/x_u = 1/2$: v changes to keep y fixed.

3.1 $\det DF = 4(u^2 + v^2) > 0$ away from zero, so the inverse-function theorem supplies a local inverse at each such point. But $F(u, v) = F(-u, -v)$, so it is not globally injective on the punctured plane. A vanishing determinant at zero means this theorem cannot guarantee a differentiable local inverse there. It is not, by itself, a general proof that no local set-theoretic inverse can exist for any map; for example the scalar cube map is bijective despite derivative zero at zero.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.