

25.1 Integrating factors and nonhomogeneous differential equations

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS integrating factor • particular solution

Objective

Solve first-order linear equations using an integrating factor; Construct the homogeneous and particular parts of a constant-coefficient solution; Handle resonance and verify the result in the original equation.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

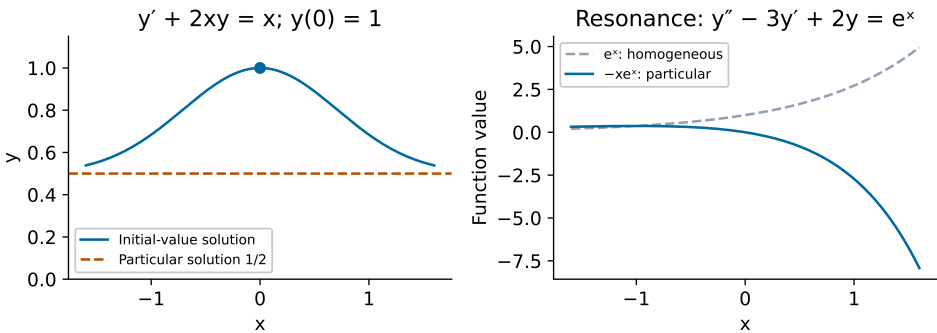
1 Key knowledge and worked examples

integrating factor : A multiplier that turns a first-order linear equation into a product derivative.

particular solution : One solution supplying the specified nonhomogeneous forcing.

A first-order linear equation has the form $y' + p(x)y = q(x)$ on an interval where its coefficients are continuous. It need not be separable. Set $\mu(x) = \exp(\int p(x)dx)$. Since $\mu' = p\mu$, multiplying gives $(\mu y)' = \mu q$, hence $y = \mu^{-1}(\int \mu q \, dx + C)$. Choose any convenient antiderivative for p ; its integration constant only rescales μ and cancels from the solution. Apply initial data after integration, and keep any singular coefficient points outside the chosen interval.

For $y'' - 3y' + 2y = 4$, the characteristic roots are 1 and 2. A constant particular solution $y_p = 2$ gives $2y_p = 4$, so $y = C_1 e^x + C_2 e^{2x} + 2$. For $y' + 2xy = x$ and $y(0) = 1$, the separate first-order solution is $y = (1 + e^{-x^2})/2$. These illustrate distinct methods; neither equation becomes homogeneous just because its left side is linear.



Original course illustration; its values belong to the worked example, not the later practice.

2 Practice

2.1 Which integrating factor works for $y' + 3y = x$? [1]

- A x^3
- B $e^{(3x)}$
- C $e^{(-3x)}$
- D $3x$
- E $1/x$

2.2 For $y'' - 3y' + 2y = 4$, which constant is a particular solution? [1]

- A -4
- B -2
- C 0
- D 2
- E 4

2.3 Solve $y' + 2y = e^x$, $y(0) = 1$, by an integrating factor. Check the answer directly. [4]

3 Exam-style questions

3.1 Solve $y'' - 2y' + y = e^x$ with $y(0) = 0, y'(0) = 0$. Explain why neither Ke^x nor Kxe^x is a sufficient particular trial. [4]
