

Solutions

2.1 B. $p(x)=3$, so $\mu=\exp(\int 3 \, dx)=e^{3x}$. Its derivative is 3μ . A valid integrating factor has logarithmic derivative three. x^3 , $3x$ and $1/x$ do not; e^{-3x} has the opposite logarithmic derivative.

2.2 D. Both derivatives vanish for a constant. The equation reduces to $2y=4$, so $y=2$. For constants the derivatives vanish and $2y=4$. Minus four, minus two, zero and four fail this substitution.

2.3 The coefficients are continuous everywhere. The integrating factor is $\mu = e^{2x}$. Multiplication gives $(e^{2x}y)' = e^{3x}$. Integration gives $y = e^x/3 + Ce^{-2x}$. The initial condition yields $C=2/3$, so $y = e^x/3 + 2e^{-2x}/3$. Differentiation gives $y' = e^x/3 - 4e^{-2x}/3$; adding $2y$ gives e^x , and $y(0)=1$.

3.1 The homogeneous root is 1 with multiplicity two, so $y_h = (C_1 + C_2x)e^x$. Both suggested trials already belong to that homogeneous space and are annihilated. Put $y = e^xv$; then $y'' - 2y' + y = e^xv''$. Therefore $v'' = 1$ and $v = x^2/2 + C_2x + C_1$. The two initial conditions give $C_1 = C_2 = 0$, hence $y = x^2e^x/2$. This substitution also checks the forcing without relying only on initial values.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.