

Solutions

2.1 C. AP corresponds to AC, not AB. The common scale is $6/15=2/5$; multiply $CB=20$ by it. The correspondence fixes scale $6/15$. Multiplying twenty gives eight; four, six, twelve and fifty use an unmatched side or wrong operation.

2.2 C. The new area is $1.3^2=1.69$ times the old area, so the increase is 69%, not the full new ratio 169%. Thirty percent uses length change directly; sixty drops the square term; ninety has no area-scale justification; 169 percent is the new percentage of the original, not the increase.

2.3 Taking positive square roots gives length ratio 3:5. Cubing gives volume ratio 27:125. The area ratio is not itself a length scale. Corresponding vertices must match equal angles; otherwise ratios can compare unrelated sides. Similarity is an assumption here; area ratios alone do not prove that arbitrary solids are similar.

3.1 Write $r_A = \sqrt{(x+5)^2 + y^2}$ and $r_B = \sqrt{(x-5)^2 + y^2}$. The original condition is $r_A - r_B = 6$. Since $r_A^2 - r_B^2 = 20x$, multiplying the distance difference by their sum gives $r_A + r_B = 10x/3$. Hence $r_B = 5x/3 - 3$, which must be nonnegative. Squaring this equality gives $(x-5)^2 + y^2 = (5x/3 - 3)^2$. Expanding and collecting gives $16x^2/9 - y^2 = 16$. Thus the half focal distance is $c=5$, half difference is $a=3$, and the hyperbola is $x^2/9 - y^2/16 = 1$, using $b^2 = c^2 - a^2 = 16$. The original positive difference selects the right branch $x \geq 3$. At $(3, 0)$ the distances are 8 and 2, difference six; at $(-3, 0)$ the difference is negative six. Squaring alone retains this invalid branch. A difference of 12 exceeds $|AB| = 10$ and is impossible by the reverse triangle inequality.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.