

Solutions

2.1 B. An output of an input in the union comes from at least one subset. Intersection equality can fail when distinct inputs share an output. A and D fail for a constant map on distinct inputs. C and E compare domain subsets with outputs without any identity-map or containment assumption.

2.2 C. Failure of the implication requires a true premise and a false conjunction. De Morgan changes the latter into a disjunction. A makes the premise false; B is not the negation of the original implication; D allows a false premise. E requires both conclusions to fail, though failure of either is enough.

2.3 If $y = f(x)$ for $x \in A \cap B$, then x belongs to each set, so y belongs to each image. For strictness, map distinct a, b to the same c and take $A = \{a\}$, $B = \{b\}$. The left image is empty and the right intersection is $\{c\}$. If f is injective and $y = f(a) = f(b)$ with $a \in A, b \in B$, injectivity gives $a=b$ in the intersection, proving reverse inclusion. Conversely the singleton example shows that equality for all pairs forces injectivity.

3.1 Reflexivity uses $a - a = 0$; symmetry uses the negative of a multiple of four; transitivity uses the sum of two such multiples. Classes are the four residues modulo four; the class of a is $a + 4\mathbb{Z}$. The negation is: there exists an integer n such that every integer m greater than n fails P . Equivalently $\exists n \forall m (m > n \Rightarrow \neg P(m))$. It does not assert that P fails for every integer, nor that m is at most n for every m .

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.