

Solutions

2.1 B. The absolute Jacobian of $(r \cos(\theta), r \sin(\theta))$ is r . Unit scale, r squared, $\sin \theta$ and inverse r do not equal the polar area Jacobian r . The sine factor belongs to spherical coordinates with different variables.

2.2 D. Divergence is 3 and unit-ball volume is $4\pi/3$, so outward flux is 4π . Divergence three times ball volume 4π over three is 4π . Twelve π uses sphere area in place of volume; zero and the smaller π multiples omit part of the flux.

2.3 The annulus has polar bounds $1 \leq r \leq 2$, $0 \leq \theta \leq 2\pi$. Including the Jacobian,

$$I = \int_0^{2\pi} \int_1^2 r^3 dr d\theta = 15\pi/2.$$

F is continuously differentiable on the whole ball and has divergence three. The divergence theorem gives outward flux $3(4\pi/3) = 4\pi$. Inward orientation would reverse its sign.

3.1 Parameterise $r(t) = (\cos t, \sin t)$ for $0 \leq t \leq 2\pi$. Then $G(r(t)) = (-\sin t, \cos t) = r'(t)$, so $G \cdot r' = 1$ and the integral is 2π . The field is undefined at the origin inside the disk. Green's theorem requires the needed smoothness on a neighbourhood of the region, not just on its boundary. Zero curl on a punctured domain does not establish a global potential; a closed nonexact field can circulate around its hole.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.