

Solutions

2.1 C. $D=2(-2)-0^2=-4$, so the point is a saddle. Minimum and maximum conflict with the negative Hessian determinant; inconclusive applies when that determinant is zero. The premise explicitly supplies a critical point.

2.2 D. $(x-y)^2 \geq 0$ implies $2xy \leq x^2 + y^2 = 2$. Equality occurs at $(1,1)$ and $(-1,-1)$. Negative two and negative one cannot be the maximum because $(1,1)$ gives one. Zero misses positive values; two violates $2xy \leq 2$.

2.3 For f the Hessian is $\text{diag}(2, -2)$ with determinant -4 ; the origin is a saddle. Along the axes the values have opposite signs. For g the Hessian at zero is zero, so the second-derivative test is inconclusive. Nevertheless $x^4 + y^4 \geq 0$, with equality only at the origin; it is a strict global minimum. An inconclusive test is not a classification of the point.

3.1 The circle is compact and its constraint gradient $(2x, 2y)$ never vanishes there. Multipliers give $y = 2\lambda x$, $x = 2\lambda y$. Neither coordinate can be zero at a stationary point; hence $y = \pm x$. The four candidates $(1, 1)$, $(-1, -1)$, $(1, -1)$, $(-1, 1)$ give global maximum 1 and minimum -1. The bound $2|xy| \leq x^2 + y^2 = 2$ checks completeness. On the line, $x^2 + (4-x)^2 = 2(x-2)^2 + 8$, so the minimum is 8 at $(2, 2)$. There is no maximum because the expression is unbounded. Multiplier candidates alone do not establish compactness.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.