

Solutions

2.1 C. The lower-bound power limit is finite when $1-p>0$; $p=1$ gives logarithmic divergence. Conditions $p>1$ and $p\geq 1$ belong to a different endpoint or include logarithmic divergence. $p\leq 1$ wrongly includes $p=1$; all real p includes divergent examples.

2.2 D. Disk radius is x^2 , so $V=\pi \int_0^1 x^4 dx=\pi/5$. Disk radius is x squared and disk area πx to the fourth. The other fractions result from integrating the wrong power or wrong geometry.

2.3 Split at 1. Near zero, convergence requires $p < 1$; near infinity, it requires $p > 1$. No real p meets both, and $p=1$ diverges logarithmically at both ends. Both one-sided parts must converge independently. For $1/x$ across zero, the negative and positive parts diverge separately, so the ordinary improper integral diverges. Symmetric cutoffs cancel to zero, yielding a principal value only. The zero is not the value of the ordinary integral.

3.1 Disks give $V = \pi \lim_{b \rightarrow \infty} \int_1^b x^{-2} dx = \pi \lim(1-1/b) = \pi$. For surface area, $y' = -1/x^2$ and

$$S = 2\pi \int_1^\infty x^{-1} \sqrt{1+x^{-4}} dx.$$

Its nonnegative integrand is at least $2\pi/x$, whose integral diverges. Comparison proves infinite lateral area. This includes neither an end cap nor a claim that finite volume implies finite area.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.