

Solutions

2.1 D. The closure is $[0,1]$ and the interior is $(0,1)$; removing the latter leaves both endpoints. Empty omits both endpoints; the singleton options omit one endpoint; $[0,1]$ incorrectly includes interior points.

2.2 C. An open interval is connected but not closed in $\mathbb{R}$, so it is not compact. The other options are compact or disconnected. The finite two-point set and the disjoint closed intervals are disconnected; the closed interval and the empty set are compact. The empty set is connected under the usual convention but still fails noncompactness.

2.3 $A = X \cap (-1, 1)$ is open in X , so its relative interior is A . Its closure is $[0, 1]$ and its boundary is $\{1\}$. Zero is interior relative to X because a sufficiently small ball in X has no negative part. A is not closed in X : the sequence $1 - 1/n$ for $n \geq 2$ tends to the missing point 1. In the ambient real line, interior and boundary would instead be $(0, 1)$ and $\{0, 1\}$.

3.1 The finite set $\{0, 2\}$ is compact but disconnected. The interval $(0, 1)$ is connected but not compact. The continuous identity function on $(0, 1)$ attains neither a maximum nor a minimum. Connectedness makes a continuous real image an interval and therefore supplies intermediate values. Compactness instead makes a nonempty continuous real image attain its extreme values. Neither property substitutes for the other.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.