

Solutions

2.1 D. $\gcd(6,18)=6$ divides 12, so there are six solutions; after dividing the modulus too, $x \equiv 2 \pmod{3}$. The six residues are 2,5,8,11,14,17. Zero, one, three and twelve are therefore incorrect counts; reducing the modulus does not leave only one residue modulo eighteen.

2.2 D. Among 1,4,7,10,13 modulo 15, only 7 has residue 2 modulo 5. Two fails the first congruence; four and eleven fail the second; five fails both. Seven has both required residues.

2.3 The gcd is four and divides 12. Dividing all three numbers gives $2x \equiv 3 \pmod{5}$, so $x \equiv 4 \pmod{5}$. Modulo 20 the four answers are 4, 9, 14, 19; substitution gives remainder 12 each time. Since $7 \cdot 3 = 21$, the inverse of 7 is 3. Although $\phi(20) = 8$, Euler's theorem requires a unit. The gcd of 2 and 20 is two, and $2^8 = 256 \equiv 16$, not 1.

3.1 The residues agree modulo the gcd three. Put $x = 2 + 6k$: then $6k \equiv 3 \pmod{9}$, hence $2k \equiv 1 \pmod{3}$ and $k \equiv 2 \pmod{3}$. Thus $x \equiv 14 \pmod{18}$. The modulus is the lcm, not 54. For the divisibility problem, write prime exponents in n . They must satisfy $3a \geq 5$, $3b \geq 7$, so $a \geq 2$, $b \geq 3$. The least n is $2^2 \cdot 3^3 = 108$; other prime factors can only increase it.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.