

Solutions

2.1 B. Left and right derivatives are -1 and 1 . A fails because one-sided derivatives differ; C fails because the function value and limit are both zero.

2.2 B. The harmonic series diverges; the term test alone is insufficient. A confuses a necessary condition with a sufficient one; C fails because there is a term for every positive integer.

2.3 The integrand is continuous on the real line. The fundamental theorem and chain rule apply at both bounds:

$$F'(x) = (1 + x^4)2x - (1 + x^2).$$

$$F'(2) = (1 + 2^4)2 \cdot 2 - (1 + 2^2) = 63.$$

Independently, an antiderivative is $t + t^3/3$. Thus $F = x^2 + x^6/3 - x - x^3/3$, with the same derivative.

3.1 Rewrite each term as $\frac{1}{n} \frac{k/n}{1+(k/n)^2}$. The continuous function $f(t) = t/(1+t^2)$ on $[0, 1]$ and mesh $1/n$ give

$$L = \int_0^1 \frac{t}{1+t^2} dt = \frac{1}{2} \ln 2.$$

The harmonic series diverges: the block from $2^{j-1} + 1$ through 2^j contains 2^{j-1} terms at least $1/2^j$, so each block adds at least $1/2$. Terms tending to zero is necessary but insufficient. The finite-sum integrand here depends on the scaled index; it is not the harmonic series.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.