

## 22.1 Similarity, scaling and conic distance loci

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 10 marks

KEY WORDS similarity • focus

### Objective

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Match triangle vertices by equal angles before forming a side ratio; Apply length, area and volume scale factors to geometric changes; Identify conic distance conditions and retain a signed hyperbola branch.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

### 1 Key knowledge and worked examples

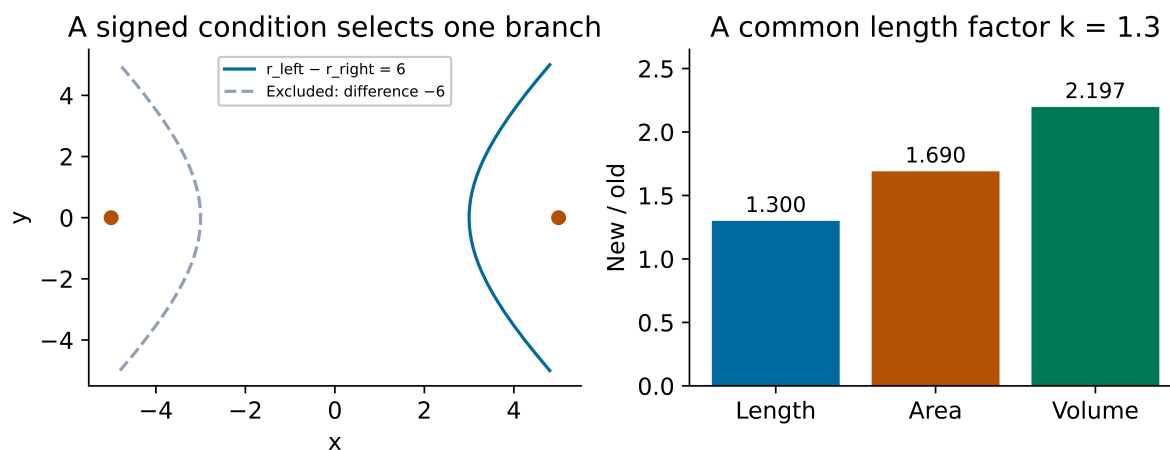
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**similarity** : Equality of corresponding angles and a common ratio of corresponding lengths.

**focus** : A fixed point used in a conic distance definition.

Triangles are similar when their corresponding angles agree; their corresponding side ratios are then equal. Write the vertex correspondence explicitly. If triangle APQ has angle A equal to angle A of ABC and angle P equal to angle C, its ordered correspondence is  $A \leftrightarrow A$ ,  $P \leftrightarrow C$ ,  $Q \leftrightarrow B$ . Thus  $AP/AC = AQ/AB = PQ/CB$ . The side AP lies on AB but corresponds to AC; matching by where a side lies or by the letter P can give the wrong ratio.

Let  $AB=10$ ,  $AC=15$  and  $BC=20$ . A point P on AB has  $AP=6$ , and Q on AC makes angle APQ equal to angle ACB. Then  $AP/AC=6/15=2/5$ , so  $PQ=8$  and  $AQ=4$ . Separately, foci at  $(-5,0)$ ,  $(5,0)$  and distance-to-left minus distance-to-right equal to 6 give  $a=3$ ,  $b^2=25-9=16$ :  $x^2/9 - y^2/16 = 1$  with  $x \geq 3$ . The point  $(-3,0)$  satisfies the squared equation but fails the signed condition.



*Original course illustration; its values belong to the worked example, not the later practice.*

## 2 Practice

**2.1** Correspondence is  $A \leftrightarrow A$ ,  $P \leftrightarrow C$ ,  $Q \leftrightarrow B$ . If  $AP=6$ ,  $AC=15$  and  $CB=20$ , what is  $PQ$ ?<sup>[1]</sup>

- A 4
- B 6
- C 8
- D 12
- E 50

**2.2** A circle radius grows by 30%. What is the area increase?

<sup>[1]</sup>

- A 30%
- B 60%
- C 69%
- D 90%
- E 169%

**2.3** Similar solids have surface-area ratio 9:25. Find their corresponding length and volume ratios. Explain why matching vertices matters before forming a triangle ratio.<sup>[4]</sup>

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### 3 Exam-style questions

**3.1** For foci  $A = (-5, 0)$  and  $B = (5, 0)$ , find the locus  $|PA| - |PB| = 6$ . Derive its standard equation, choose its branch and check the vertex in the unsquared relation. What happens if the requested difference is 12? <sup>[4]</sup>

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## Solutions

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**2.1 C.** AP corresponds to AC, not AB. The common scale is  $6/15=2/5$ ; multiply  $CB=20$  by it. The correspondence fixes scale  $6/15$ . Multiplying twenty gives eight; four, six, twelve and fifty use an unmatched side or wrong operation.

**2.2 C.** The new area is  $1.3^2=1.69$  times the old area, so the increase is 69%, not the full new ratio 169%. Thirty percent uses length change directly; sixty drops the square term; ninety has no area-scale justification; 169 percent is the new percentage of the original, not the increase.

**2.3** Taking positive square roots gives length ratio 3:5. Cubing gives volume ratio 27:125. The area ratio is not itself a length scale. Corresponding vertices must match equal angles; otherwise ratios can compare unrelated sides. Similarity is an assumption here; area ratios alone do not prove that arbitrary solids are similar.

**3.1** Write  $r_A = \sqrt{(x+5)^2 + y^2}$  and  $r_B = \sqrt{(x-5)^2 + y^2}$ . The original condition is  $r_A - r_B = 6$ . Since  $r_A^2 - r_B^2 = 20x$ , multiplying the distance difference by their sum gives  $r_A + r_B = 10x/3$ . Hence  $r_B = 5x/3 - 3$ , which must be nonnegative. Squaring this equality gives  $(x-5)^2 + y^2 = (5x/3 - 3)^2$ . Expanding and collecting gives  $16x^2/9 - y^2 = 16$ . Thus the half focal distance is  $c=5$ , half difference is  $a=3$ , and the hyperbola is  $x^2/9 - y^2/16 = 1$ , using  $b^2 = c^2 - a^2 = 16$ . The original positive difference selects the right branch  $x \geq 3$ . At  $(3, 0)$  the distances are 8 and 2, difference six; at  $(-3, 0)$  the difference is negative six. Squaring alone retains this invalid branch. A difference of 12 exceeds  $|AB| = 10$  and is impossible by the reverse triangle inequality.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.