

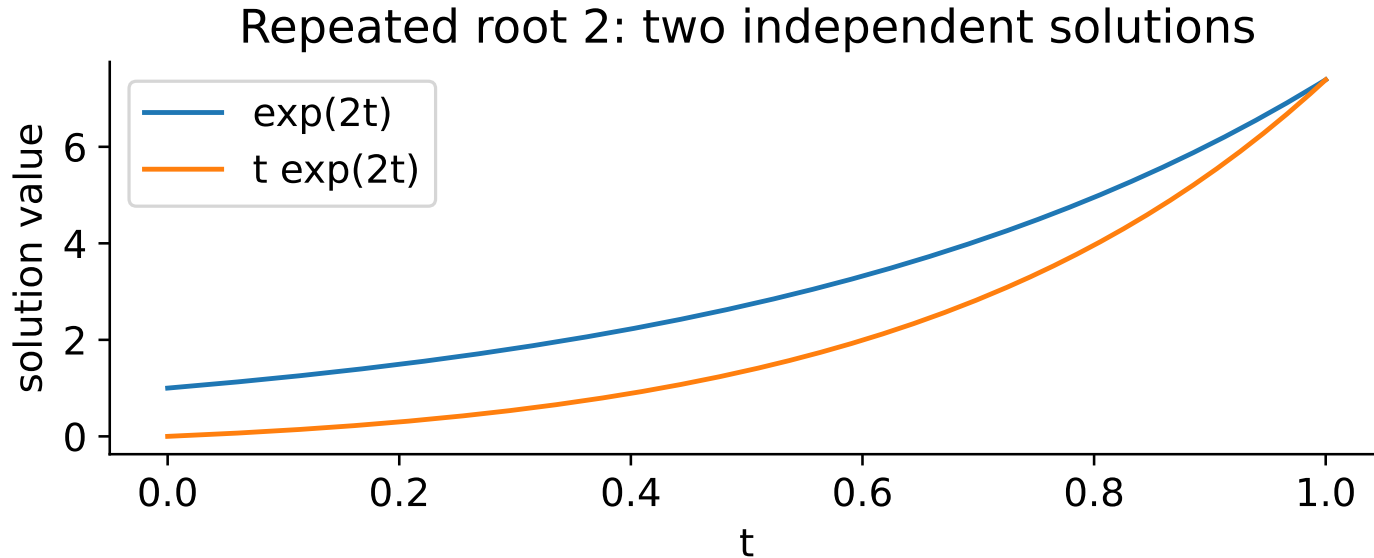
Exercise walk-through

8.1

Differential equations and initial conditions

GRE · GRE Subject Test · GRE Mathematics

Figure 11: Worked reasoning



Check the labelled values and conditions.

14. Reasoning and check

Reasoning and check

The characteristic polynomial is $(r - 2)^2$. Independent solutions are e^{2t} and te^{2t} , hence $y = (C_1 + C_2 t)e^{2t}$. The initial value gives $C_1 = 1$; the derivative at zero gives $2C_1 + C_2 = 0$, so $C_2 = -2$.

$y(t) = (1 - 2t)e^{2t}$, $y'(t) = -4te^{2t}$, $y''(t) = (-4 - 8t)e^{2t}$. Substitution gives $y'' - 4y' + 4y = 0$, and the two initial values hold. Two copies of e^{2t} would not span the solution space.

16. Reasoning and check

Reasoning and check

One solution is identically zero. For any $a \geq 0$, set $y_a(t) = 0$ for $0 \leq t \leq a$ and $y_a(t) = (t - a)^2$ for $t > a$. The derivative is zero on the first interval and $2(t - a)$ on the second; both sides have derivative zero at a . Therefore y_a is differentiable and satisfies the equation and initial value. The right side is continuous but is not locally Lipschitz in y at zero: $|2\sqrt{h} - 0|/|h| = 2/\sqrt{h}$ is unbounded. Existence does not imply uniqueness, and division by $\sqrt{y}$ would lose the waiting-time solutions.

Exercises 2.1·2.2

2.1 For $y'=2y$ and $y(0)=3$, the solution is:

[1]

- **A** $3e^{(2t)}$
- **B** $2e^{(3t)}$
- **C** $3+2t$

2.2 For a repeated root $r=-1$, the general solution includes:

[1]

- **A** $C_1e^{(-t)}+C_2e^{(-t)}$
- **B** $(C_1+C_2t)e^{(-t)}$
- **C** $C_1 \cos t+C_2 \sin t$

Exercises 2.1·2.2

2.1 For $y'=2y$ and $y(0)=3$, the solution is: [1]

- **A** $3e^{(2t)}$
- **B** $2e^{(3t)}$
- **C** $3+2t$

2.1 A. The initial condition fixes $C=3$. The second exponential has wrong initial value and rate; the linear expression has derivative two but twice its value is not constantly two.

2.2 For a repeated root $r=-1$, the general solution includes: [1]

- **A** $C_1e^{(-t)}+C_2e^{(-t)}$
- **B** $(C_1+C_2t)e^{(-t)}$
- **C** $C_1 \cos t+C_2 \sin t$

2.2 B. The factor t supplies a second independent solution. Two multiples of the same exponential combine into one solution. Sine and cosine satisfy the characteristic equation $r^2+1=0$, not a repeated root minus one.

Exercise 2.3

2.3 Solve $y'' - 4y' + 4y = 0$ with $y(0) = 1$, $y'(0) = 0$. Check both initial conditions and

3 Exam-style questions

Exercise 2.3

2.3 Solve $y'' - 4y' + 4y = 0$ with $y(0) = 1$, $y'(0) = 0$. Check both initial conditions and

3 Exam-style questions

2.3 The characteristic polynomial is $(r - 2)^2$. Independent solutions are e^{2t} and te^{2t} , hence $y = (C_1 + C_2t)e^{2t}$. The initial value gives $C_1 = 1$; the derivative at zero gives $2C_1 + C_2 = 0$, so $C_2 = -2$.

$$y(t) = (1 - 2t)e^{2t}, \quad y'(t) = -4te^{2t}, \quad y''(t) = (-4 - 8t)e^{2t}.$$

Substitution gives $y'' - 4y' + 4y = 0$, and the two initial values hold. Two copies of e^{2t} would not span the solution space.

Exercise 3.1

3.1 Show that the initial-value problem $y' = 2\sqrt{|y|}$, $y(0) = 0$ has more than one solution for $t \geq 0$. Identify the missing standard uniqueness hypothesis. [4]

Exercise 3.1

3.1 Show that the initial-value problem $y' = 2\sqrt{|y|}$, $y(0) = 0$ has more than one solution for $t \geq 0$. Identify the missing standard uniqueness hypothesis. [4]

3.1 One solution is identically zero. For any $a \geq 0$, set $y_a(t) = 0$ for $0 \leq t \leq a$ and $y_a(t) = (t - a)^2$ for $t > a$. The derivative is zero on the first interval and $2(t - a)$ on the second; both sides have derivative zero at a . Therefore y_a is differentiable and satisfies the equation and initial value. The right side is continuous but is not locally Lipschitz in y at zero: $|2\sqrt{h} - 0|/|h| = 2/\sqrt{h}$ is unbounded. Existence does not imply uniqueness, and division by $\sqrt{y}$ would lose the waiting-time solutions.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.