

Exercise walk-through

6.1

Discrete mathematics, probability and numerical methods

GRE · GRE Subject Test · GRE Mathematics

7. Choose and justify a method

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Use complements to count at least one occurrence, and condition on the actual remaining population after a draw without replacement. For r iid outcomes chosen from n equally likely values, the probability that all are distinct is $n(n-1)\cdots(n-r+1)/n^r$ when $r \leq n$. Its complement counts repeated outcomes. For a binomial count with N independent trials and fixed success probability p , mean is Np and variance $Np(1-p)$. Identical probabilities alone do not establish independence.

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Numerical approximation needs an error argument. Bisection preserves a sign-changing bracket for a continuous function and halves its width at each step; a zero may be absent if continuity fails. Newton's update is $x_{\text{new}} = x - f(x)/f'(x)$, requiring a nonzero derivative at the current point; convergence is not automatic from every starting value. An approximation's residual and its error in x are different. State the method's assumptions and a stopping criterion, rather than treating extra displayed decimals as accuracy.

14. Reasoning and check

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There are 5^4 equally likely ordered draw strings. The no-repeat strings number $5 \cdot 4 \cdot 3 \cdot 2 = 120$. $P(\text{repeat}) = 1 - \frac{5 \cdot 4 \cdot 3 \cdot 2}{5^4} = \frac{101}{125}$. Ordered distinct selections number 120; unordered subsets number $\binom{5}{4} = 5$. Each subset occurs in $4! = 24$ orders. Dividing by $4!$ is appropriate only for the unordered count, not the draw probability denominator.

Exercises 2.1·2.2

2.1 How many unordered pairs can be chosen from 5 objects?

[1]

- A 10
- B 20
- C 25

2.2 A tree has 8 vertices. Its edge count is:

[1]

- A 7
- B 8
- C 9

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2.1 A. $5 \text{ choose } 2 = 5 \times 4 / 2 = 10$. Twenty counts ordered distinct pairs, counting each twice. Twenty-five permits repeated objects and order.

2.2 A tree has 8 vertices. Its edge count is:

[1]

- A 7
- B 8
- C 9

2.2 A. A finite tree has $n-1$ edges. Eight or nine edges would force a cycle in a connected graph with eight vertices; a finite tree has exactly seven.

Exercise 2.3

2.3 Four labelled draws are independent and uniform from five symbols, with replacement. Find the probability that at least two draws agree. Compare the number of ordered and unordered selections of four distinct symbols. [4]

3 Exam-style questions

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3.1 The polynomial is continuous; $f(1) < 0 < f(2)$. The first midpoint is 1.5 with positive value, retaining $[1, 1.5]$. The next is 1.25 with negative value, retaining $[1.25, 1.5]$. After ten halvings, width is 2^{-10} and the midpoint of that retained bracket differs from the bracketed root by at most $2^{-11} = 1/2048$. The error bound uses the bracket, not a small residual alone. For $1/x$, zero is a singularity and there is no root. Continuity on the bracket fails; opposite endpoint signs do not justify the method.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.