

Exercise walk-through

24.1

Vector geometry, projections and oriented area

GRE · GRE Subject Test · GRE Mathematics

7. Choose and justify a method

Choose and justify a method

For three-dimensional vectors, $\mathbf{u} \times \mathbf{v}$ is perpendicular to both with length $|\mathbf{u}||\mathbf{v}| \sin \theta$. Coordinate calculation uses $(u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$. This length is the parallelogram area, so a triangle from two edge vectors has half that area. Reversing their order reverses the cross product but preserves area. Build both edge vectors from the same vertex; crossing two unrelated position vectors generally measures the wrong triangle.

14. Reasoning and check

Reasoning and check

$v \cdot u = 14$ and $u \cdot u = 5$, so the vector projection is $(14/5)u = (14/5, 0, 28/5)$. The residual is $(-4/5, 3, 2/5)$, whose dot product with u is zero. The cross product is $u \times v = (-6, -2, 3)$ with norm seven. Triangle area is half the parallelogram area, or $7/2$. Both edge vectors start at O ; crossing unrelated position vectors would not generally compute the requested triangle.

16. Reasoning and check

Reasoning and check

Order their directions around the circle. Every consecutive gap must exceed $\pi/2$: if a gap were at most $\pi/2$, that pair would have nonnegative dot product. Four such gaps would sum to more than 2π , impossible. Therefore not all six products can be strictly negative. With nonpositive products, $e_1, e_2, -e_1, -e_2$ works: consecutive pairs are perpendicular and opposite pairs have negative products. A zero vector cannot satisfy strict negativity.

Exercise 2.1

2.1 Triangle vertices are $(0,0,0), (2,0,0), (0,3,0)$. What is its area?

[1]

- A 1
- B 2
- C 3
- D 6
- E 12

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- A 1
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- D 6
- E 12

2.1 C. The edge cross product has magnitude 6. Triangle area is half the spanned parallelogram area. The cross-product norm is six, so six is the parallelogram area and twelve doubles it; one and two fail the same half-base-times-height calculation.

Exercises 2.2·2.3

2.2 What is the vector projection of $(3,4)$ onto the line spanned by $(1,0)$? [1]

- **A** $(0,4)$
- **B** $(3,0)$
- **C** $(1,0)$
- **D** $(3/5,4/5)$
- **E** 5

2.3 Project $v = (2, 3, 6)$ onto $u = (1, 0, 2)$. Find the area of the triangle with vertices O, u and v. Check the projection residual is orthogonal to u. [4]

Exercises 2.2-2.3

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- C $(1,0)$
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2.2 B. Multiply $(1,0)$ by the coefficient $((3,4) \cdot (1,0))/((1,0) \cdot (1,0))=3$. $(0,4)$ is the orthogonal residual; $(1,0)$ is a direction rather than the projected length; the unit version of v projects onto no stated line; five is a scalar norm, not a vector projection.

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Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.