

Exercise walk-through

23.1

Trigonometric phase and parametric curves

GRE · GRE Subject Test · GRE Mathematics

6. Choose and justify a method

Choose and justify a method

For $y=A \cos(\omega x-\phi)$ with $A>0$ and $\omega>0$, amplitude is A , period is $2\pi/\omega$, and horizontal shift is ϕ/ω . The phase angle ϕ is measured inside the cosine argument; it is not itself the horizontal shift unless $\omega=1$. Start with peak-to-peak spacing for the period and midline-to-peak distance for amplitude. To find phase, substitute a known point and check the direction of motion there. Equivalent phases differ by 2π .

7. Choose and justify a method

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For $y = -2 \cos(3x)$, rewrite the model with positive amplitude as $2 \cos(3x - \pi)$. Its amplitude is 2, period $2\pi/3$, and phase π ; the equivalent right shift is $\pi/3$. Peaks occur where $3x - \pi$ is a multiple of 2π . A graph point at $x=0$ and $y=0$ alone cannot determine phase uniquely: slopes or another point distinguish the possible angles. Units and angle conventions matter; ordinary calculus trigonometric derivatives use radians.

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A parametric curve specifies $x=x(t)$, $y=y(t)$. Eliminating t describes a point set but can lose restrictions and direction. For $x=\cos^3 t$, $y=\sin^3 t$, real cube roots give $|x|^{2/3}+|y|^{2/3}=1$, an astroid with cusps on the axes. As t runs from 0 to 2π it starts at $(1,0)$, passes $(0,1)$ at $\pi/2$ and travels counterclockwise. Restricting t to $[0,\pi/2]$ gives only the first-quadrant arc, not the entire implicit locus.

Exercise 2.1

2.1 For $y=2 \cos(3x-\pi)$, what is the rightward horizontal shift?

[1]

- **A** π
- **B** 3π
- **C** $\pi/3$
- **D** $2\pi/3$
- **E** $1/3$

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2.1 C. Factor the argument as $3(x-\pi/3)$. The phase π is divided by angular frequency 3. Pi is the phase before division; three pi multiplies rather than divides; two pi over three is the period, and one-third omits the phase pi.

Exercises 2.2·2.3

2.2 For $x=\cos^3 t, y=\sin^3 t$ with t in $[0, \pi/2]$, which set is traced?

[1]

- **A** The whole unit circle
- **B** The whole astroid
- **C** Only the first-quadrant astroid arc
- **D** Only the line $x+y=1$
- **E** A circle of radius 3

2.3 For $y = -3\cos(2x)$, state amplitude, period, a positive-amplitude phase representation and the corresponding horizontal shift. Parameterise the ellipse $x^2/9 + y^2/4 = 1$ counterclockwise.

[4]

Exercises 2.2-2.3

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2.2 C. The parameter interval keeps both coordinates nonnegative. Elimination gives the astroid relation, but does not remove this quadrant restriction. The full curves exceed the parameter interval; the circle and line have different equations. The restricted first-quadrant arc alone matches both coordinate signs and the astroid relation.

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2.3 Amplitude is three and period $2\pi/2 = \pi$. Write $y = 3\cos(2x - \pi)$, with phase π and rightward shift $\pi/2$. Phase and shift are not the same. One parametrisation is $x = 3\cos t, y = 2\sin t, 0 \leq t \leq 2\pi$, which begins at $(3, 0)$ and initially moves upward. Eliminating t checks the ellipse equation.

Exercise 3.1

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3.1 Elimination gives $y^2 = x^3$, with $0 \leq x \leq 1$ and both signs of y . For t nonzero, $dy/dx = 3t/2 \rightarrow 0$. The secant slope from the origin is $y/x = t \rightarrow 0$ too, so the cusp has horizontal tangent. Both parameter derivatives vanish at zero, making the raw quotient $0/0$ inconclusive, not proving absence of a tangent. The unrestricted implicit equation also has points with $x > 1$; the interval restriction must be carried separately.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.