

Exercise walk-through

22.1

Similarity, scaling and conic distance loci

GRE · GRE Subject Test · GRE Mathematics

8. Choose and justify a method

Choose and justify a method

A circle fixes distance to one point. An ellipse fixes the sum of distances to two foci. A hyperbola fixes the absolute difference of those distances; a parabola fixes equality of distance to a focus and to a directrix. These are definitions of point sets, not sketches to memorise. For foci $(-c,0)$ and $(c,0)$, an ellipse uses a sum $2a$ with $a > c$. A nondegenerate hyperbola uses an absolute difference $2a$ with $0 < a < c$; a signed difference distinguishes its two branches.

9. Choose and justify a method

Choose and justify a method

For the hyperbola with horizontal transverse axis, $c^2 = a^2 + b^2$ and the equation is $x^2/a^2 - y^2/b^2 = 1$. Let r_A be distance to $(-c, 0)$ and r_B distance to $(c, 0)$. The condition $r_A - r_B = 2a > 0$ selects the right branch $x \geq a$; the negative condition selects the left. Squaring can lose this sign, so check the original distance condition after obtaining the equation. Difference zero gives the perpendicular bisector rather than a hyperbola; a difference exceeding the focal separation is impossible by the triangle inequality.

16. Reasoning and check

Reasoning and check

Write $r_A = \sqrt{(x+5)^2 + y^2}$ and $r_B = \sqrt{(x-5)^2 + y^2}$. The original condition is $r_A - r_B = 6$. Since $r_A^2 - r_B^2 = 20x$, multiplying the distance difference by their sum gives $r_A + r_B = 10x/3$. Hence $r_B = 5x/3 - 3$, which must be nonnegative. Squaring this equality gives $(x-5)^2 + y^2 = (5x/3 - 3)^2$. Expanding and collecting gives $16x^2/9 - y^2 = 16$. Thus the half focal distance is $c=5$, half difference is $a=3$, and the hyperbola is $x^2/9 - y^2/16 = 1$, using $b^2 = c^2 - a^2 = 16$. The original positive difference selects the right branch $x \geq 3$. At

Exercise 2.1

2.1 Correspondence is $A \leftrightarrow A$, $P \leftrightarrow C$, $Q \leftrightarrow B$. If $AP=6$, $AC=15$ and $CB=20$, what is PQ ?^[1]

- **A** 4
- **B** 6
- **C** 8
- **D** 12
- **E** 50

Exercise 2.1

2.1 Correspondence is $A \leftrightarrow A$, $P \leftrightarrow C$, $Q \leftrightarrow B$. If $AP=6$, $AC=15$ and $CB=20$, what is PQ ?^[1]

- **A** 4
- **B** 6
- **C** 8
- **D** 12
- **E** 50

2.1 C. AP corresponds to AC , not AB . The common scale is $6/15=2/5$; multiply $CB=20$ by it. The correspondence fixes scale $6/15$. Multiplying twenty gives eight; four, six, twelve and fifty use an unmatched side or wrong operation.

Exercises 2.2·2.3

2.2 A circle radius grows by 30%. What is the area increase?

[1]

- **A** 30%
- **B** 60%
- **C** 69%
- **D** 90%
- **E** 169%

2.3 Similar solids have surface-area ratio 9:25. Find their corresponding length and volume ratios. Explain why matching vertices matters before forming a triangle ratio.[4]

Exercises 2.2·2.3

2.2 A circle radius grows by 30%. What is the area increase?

[1]

- A 30%
- B 60%
- C 69%
- D 90%
- E 169%

2.2 C. The new area is $1.3^2=1.69$ times the old area, so the increase is 69%, not the full new ratio 169%. Thirty percent uses length change directly; sixty drops the square term; ninety has no area-scale justification; 169 percent is the new percentage of the original, not the increase.

2.3 Similar solids have surface-area ratio 9:25. Find their corresponding length and volume ratios. Explain why matching vertices matters before forming a triangle ratio.[4]

2.3 Taking positive square roots gives length ratio 3:5. Cubing gives volume ratio 27:125. The area ratio is not itself a length scale. Corresponding vertices must match equal angles; otherwise ratios can compare unrelated sides. Similarity is an assumption here; area ratios alone do not prove that arbitrary solids are similar.

Exercise 3.1

3.1 For foci $A = (-5, 0)$ and $B = (5, 0)$, find the locus $|PA| - |PB| = 6$. Derive its standard equation, choose its branch and check the vertex in the unsquared relation. What happens if the requested difference is 12? [4]

Exercise 3.1

3.1 For foci $A = (-5, 0)$ and $B = (5, 0)$, find the locus $|PA| - |PB| = 6$. Derive its standard equation, choose its branch and check the vertex in the unsquared relation. What happens if the requested difference is 12? [4]

3.1 Write $r_A = \sqrt{(x+5)^2 + y^2}$ and $r_B = \sqrt{(x-5)^2 + y^2}$. The original condition is $r_A - r_B = 6$. Since $r_A^2 - r_B^2 = 20x$, multiplying the distance difference by their sum gives $r_A + r_B = 10x/3$. Hence $r_B = 5x/3 - 3$, which must be nonnegative. Squaring this equality gives $(x-5)^2 + y^2 = (5x/3 - 3)^2$. Expanding and collecting gives $16x^2/9 - y^2 = 16$. Thus the half focal distance is $c=5$, half difference is $a=3$, and the hyperbola is $x^2/9 - y^2/16 = 1$, using $b^2 = c^2 - a^2 = 16$. The original positive difference selects the right branch $x \geq 3$. At $(3, 0)$ the distances are 8 and 2, difference six; at $(-3, 0)$ the difference is negative six. Squaring alone retains this invalid branch. A difference of 12 exceeds $|AB| = 10$ and is impossible by the reverse triangle inequality.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.