

Exercise walk-through

21.1

Conditioning, Bayesian inference and sampling error

GRE · GRE Subject Test · GRE Mathematics

6. Choose and justify a method

Choose and justify a method

For $P(B) > 0$, $P(A|B) = P(A \cap B) / P(B)$: restrict the population to B before computing the fraction in A . Independence means $P(A \cap B) = P(A)P(B)$, equivalently $P(A|B) = P(A)$ when the denominator is nonzero. Mutually exclusive events with positive probabilities cannot be independent, because their joint probability is zero. Sampling without replacement usually changes later probabilities; a fixed denominator for successive draws would describe a different experiment.

9. Choose and justify a method

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For independent identically distributed observations with finite variance σ^2 , the sample mean has expectation μ and variance σ^2/n ; its standard error is $\sigma/\sqrt{n}$. This is spread of repeated sample means, not the spread of individual observations. Quadrupling n halves the standard error, rather than quartering it. Normal population data give an exactly normal mean; otherwise a central-limit approximation needs adequate conditions and sample size. Correlation invalidates the simple independent variance calculation.

16. Reasoning and check

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Independence gives $\text{Var}(\bar{X}) = 25/n$ and at 100 standard error $5/10 = 0.5$. With covariance one, $\text{Var}(\bar{X}) = \frac{25n+n(n-1)}{n^2} = 1 + \frac{24}{n}$. At 100 it is 1.24. The standard error tends to one as n grows, rather than to zero; quadrupling n need not halve it. Covariance terms may be dropped only under justified uncorrelatedness, with independence a sufficient condition.

Exercise 2.1

2.1 $P(A)=0.4$, $P(B)=0.5$ and $P(A\cap B)=0.3$. What is $P(A|B)$?

[1]

- **A** 0.2
- **B** 0.3
- **C** 0.4
- **D** 0.5
- **E** 0.6

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2.1 E. Restrict to B: divide the joint probability 0.3 by $P(B)=0.5$, obtaining 0.6. The conditioned denominator is 0.5. The smaller numerical choices use a marginal, joint probability or different operation, and none equals 0.3 divided by 0.5.

Exercise 2.2

2.2 An iid sample of size 25 has mean standard error 6. What is the mean standard error for size 100 from the same population? [1]

- **A** 1.5
- **B** 3
- **C** 6
- **D** 12
- **E** 24

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- A 1.5
- B 3
- C 6
- D 12
- E 24

2.2 B. Population standard deviation is $6\sqrt{25}=30$. Dividing by $\sqrt{100}=10$ gives standard error 3. Quadrupling sample size divides standard error by two. One-point-five divides by four; six leaves it fixed; twelve and twenty-four reverse the direction.

Exercise 2.3

2.3 In a stated model, 10% of devices have a defect. A test is positive for 80% of defective and 5% of nondefective devices. Find the probability of a defect given a positive result using 1000 expected devices. [4]

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2.3 Expected counts are 100 defective and 900 nondefective. Positive counts are $100(0.8) = 80$ and $900(0.05) = 45$, respectively. The conditioned group contains 125 positives, hence

$$P(D \mid +) = \frac{80}{80 + 45} = \frac{16}{25} = 0.64.$$

Negative counts 20 and 855 check both row totals. The sensitivity 0.8 is $P(+ \mid D)$, not the requested reversed condition. These are illustrative model rates, not measured performance claims.

Exercise 3.1

3.1 Independent observations have variance 25. Find the standard error of their mean for $n=100$. If instead every distinct pair has covariance 1, find the mean's variance. Does quadrupling n necessarily halve the standard error in that second model? [4]

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At 100 it is 1.24. The standard error tends to one as n grows, rather than to zero; quadrupling n need not halve it. Covariance terms may be dropped only under justified uncorrelatedness, with independence a sufficient condition.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.