

Exercise walk-through

2.1

Multivariable calculus and vector analysis

GRE · GRE Subject Test · GRE Mathematics

6. Choose and justify a method

Choose and justify a method

A partial derivative changes one coordinate while fixing the others. For $f=x^2+3xy$, $f_x=2x+3y$ and $f_y=3x$. The gradient collects these derivatives; if f is differentiable, the directional derivative along a unit vector v is $\text{grad } f \cdot v$. Normalise the direction before taking this dot product. A direction vector of length two would double the answer if used without normalisation.

8. Choose and justify a method

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A multiple integral sums contributions over a specified region. Describe the region before choosing iterated limits; nonrectangular bounds may change when the order changes. In polar coordinates area is $r \, dr \, d\theta$, not just $dr \, d\theta$. A general coordinate change uses the absolute Jacobian determinant. Sign belongs to oriented vector quantities, while area and volume scaling use a nonnegative factor.

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If both first partials of a globally defined function on $\mathbb{R}^2$ are constant, $f_x=a$ and $f_y=b$, integrating successively gives $f=ax+by+c$, a plane. This cannot be inferred from parallel straight level sets alone: e^x has vertical parallel level lines but a curved graph. Nor do $f_{xy}=f_{yx}=0$ force a plane: x^2+y^2 is a counterexample. Distinguish first-derivative constancy from absent mixed dependence and from the shape of selected level sets.

Exercises 2.1·2.2

2.1 Partial derivative with respect to x of x^2y is:

[1]

- **A** x^2
- **B** $2xy$
- **C** $2x$

2.2 The polar area element is:

[1]

- **A** $dr d\theta$
- **B** $r dr d\theta$
- **C** $r^2 dr d\theta$

Exercises 2.1·2.2

2.1 Partial derivative with respect to x of x^2y is: [1]

- A x^2
- B $2xy$
- C $2x$

2.1 B. Treat y as constant. A differentiates the wrong factor; C drops the fixed factor y . At $y=0$ every derivative value is zero, consistently with $2xy$.

2.2 The polar area element is: [1]

- A $dr d\theta$
- B $r dr d\theta$
- C $r^2 dr d\theta$

2.2 B. The Jacobian of the polar transformation is r . A omits the local scale r ; C is the radial power for a three-dimensional spherical volume, with an additional angular factor, not a planar area element.

Exercise 2.3

2.3 For $f(x, y) = x^2 + xy + 2y^2$, find the directional derivative at $(1, -1)$ towards $(3, 4)$.
Evaluate $\iint_D (x^2 + y^2) dA$ for the unit disk D . [4]

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2.3 The polynomial is differentiable. Normalise the direction: $u = (3/5, 4/5)$. The gradient is $(2x + y, x + 4y)$, hence $(1, -3)$ at the point.

$$D_u f(1, -1) = \nabla f(1, -1) \cdot u = 1 \cdot 3/5 - 3 \cdot 4/5 = -9/5.$$

Polar coordinates give $x^2 + y^2 = r^2$ and $dA = r dr d\theta$:

$$I = \int_0^{2\pi} \int_0^1 r^3 dr d\theta = \pi/2.$$

Exercise 3.1

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3.1 Along each axis $g = 0$, so both partials are zero. Continuity follows from $|xy| \leq (x^2 + y^2)/2$, which gives $|g(x, y)| \leq \sqrt{x^2 + y^2}/2 \rightarrow 0$. If differentiable, its linear derivative would be zero. Along $x = y = t \neq 0$,

$$\frac{|g(t, t)|}{\sqrt{t^2 + t^2}} = \frac{1}{2}.$$

The required remainder ratio does not tend to zero. Thus continuity and existing partial derivatives do not establish differentiability.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.