

Exercise walk-through

19.1

Functions, inverse branches and composition

GRE · GRE Subject Test · GRE Mathematics

7. Choose and justify a method

Choose and justify a method

An inverse function reverses a bijection. Solve $y=f(x)$ for x , then use the original domain to choose a branch. For $f(x)=(x-2)^2-5$ on $x\geq 2$, the inverse is $2+\sqrt{y+5}$ on $y\geq -5$. The minus branch would return inputs outside the chosen domain. Check $f^{-1}(f(x))=x$ for allowed x and $f(f^{-1}(y))=y$ for allowed y ; one unchecked composition can conceal a domain error.

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The inverse graph reflects the original graph across $y=x$; it does not take reciprocals of output values. A self-inverse function, or involution, satisfies $f(f(x))=x$ wherever the composition is defined. Both $-x$ on $\mathbb{R}$ and $1/x$ on $\mathbb{R}$ excluding zero are involutions. A strictly increasing involution on an interval must be the identity: if $f(x)>x$, increasingness gives $f(f(x))>f(x)>x$, and the analogous argument rules out $f(x)<x$.

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Iteration f^n means repeated composition, not the power $(f(x))^n$. Calculate the first few compositions and check their domains before looking for a period. For $f(x)=1/(1-x)$ on $\mathbb{R}$ excluding 0 and 1, $f^2(x)=(x-1)/x$ and $f^3(x)=x$. The image stays in the same allowed domain. Therefore reduce an iteration count modulo three. A displayed formula equal to x after cancellation does not restore forbidden inputs.

Exercise 2.1

2.1 For $f:[2,\infty)\rightarrow[-5,\infty)$, $f(x)=(x-2)^2-5$, what is $f^{-1}(y)$?

[1]

- **A** $2+\sqrt{y+5}$
- **B** $2-\sqrt{y+5}$
- **C** $\sqrt{y-5}$
- **D** $1/((y-2)^2-5)$
- **E** $(y+2)^2-5$

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2.1 A. The input restriction $x\geq 2$ forces $x-2\geq 0$, so select the positive square-root branch. The negative branch violates $x\geq 2$; square root of $y-5$ has wrong shift. A reciprocal is not an inverse function, and the last quadratic does not undo the original square.

Exercises 2.2·2.3

2.2 For $g(x)=1/(1-x)$ on $\mathbb{R}$ excluding 0 and 1, what is $g^8(3)$?

[1]

- A 3
- B $-1/2$
- C $2/3$
- D $1/256$
- E 9

2.3 Restrict $f(x) = (x-1)^2 + 2$ to $x \geq 1$. State its image and inverse, including domains.

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2.2 C. The orbit is 3, $-1/2$, $2/3$, then 3 again. Eight is congruent to two modulo three. Three is the zero-mod-three iterate; minus one-half is the one-mod-three iterate. One over 256 treats composition as multiplication, and nine squares the input instead of iterating the map.

2.3 Restrict $f(x) = (x-1)^2 + 2$ to $x \geq 1$. State its image and inverse, including domains.

2.3 The image is $[2, \infty)$. Solving $y = (x-1)^2 + 2$ with $x \geq 1$ selects $x = 1 + \sqrt{y-2}$. Thus $f^{-1} : [2, \infty) \rightarrow [1, \infty)$ has that formula. For allowed x , $1 + \sqrt{(x-1)^2} = x$ because $x-1 \geq 0$. For allowed y , $(\sqrt{y-2})^2 + 2 = y$. The negative square-root branch belongs to a different restriction, not a second inverse on this domain.

Exercise 3.1

3.1 Let $h(x) = 1/(1 - x)$. Determine the domain of $h \circ h \circ h$ and its formula. May the simplified expression be extended to the excluded points without changing the original composite?

[4]

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3.1 The first stage requires $x \neq 1$. The second requires $h(x) \neq 1$, which excludes $x=0$. For $x \neq 0, 1$, $h^2(x) = (x - 1)/x$. This never equals one, so the third stage adds no exclusion. Then $h^3(x) = x$ on $\mathbb{R} \setminus \{0, 1\}$. The identity formula is defined more widely, but adding 0 or 1 defines an extension, not the original composite. Each intermediate input must be permitted before cancellation.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.