

Data analysis: spread, weighted averages and conditional probability

Original classroom teaching; not an official form or score predictor

1. Scope and task

Scope and task

Original classroom practice within the reviewed shorter-test task types. This does not simulate an adaptive test or predict a scaled or writing score. Earlier official public forms remain exposed practice; their version and writing-source holds still apply.

2. Scope and task

Scope and task

Distinguish mean, median, spread and a weighted average Interpret distributions and overlapping or conditional event counts Use the correct reference total in table-based inference

3. Scope and task

Scope and task

weighted average : An average that accounts for each group's contribution or size.

4. Scope and task

Scope and task

mutually exclusive : Events that cannot occur together in the same outcome.

5. Read the evidence and choose a method

Read the evidence and choose a method

Mean is sum divided by count; median is the central ordered value or the mean of the middle two. Range is maximum minus minimum. An extreme value can alter mean and range without changing the median. Standard deviation measures spread about the mean; adding a constant changes the centre but not spread, while multiplying all values by c multiplies standard deviation by $|c|$.

6. Read the evidence and choose a method

Read the evidence and choose a method

Combine groups with their counts, not an unweighted mean of group means unless sizes are equal. A group of 10 with mean 6 contributes total 60; a group of 30 with mean 10 contributes total 300. The combined mean is $(60+300)/40=9$. A percentage of a subgroup uses that subgroup's denominator.

7. Read the evidence and choose a method

Read the evidence and choose a method

For equally likely discrete outcomes, probability is favourable count over total count. Conditional probability restricts the denominator to the given condition. Without replacement changes later counts. Overlapping events need subtraction of the overlap when counting a union. Independence must be given or justified; it is not the same as mutually exclusive.

8. Read the evidence and choose a method

Read the evidence and choose a method

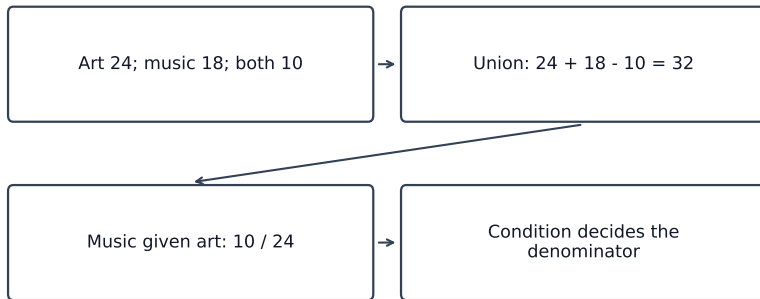
Read displays for their axes, units and sample description before making inferences. A correlation does not isolate a cause. A convenience sample may not represent the target population. Basic counting can use ordered multiplication when roles differ, or divide duplicated arrangements when order does not matter. Keep the events and outcomes explicitly defined.

9. Worked reasoning

Worked reasoning

A bag holds four red counters, three marked, and six blue counters, one marked. There are four marked counters in total. $P(\text{red})=4/10$, $P(\text{marked})=4/10$, $P(\text{red given marked})=3/4$. $P(\text{two red without replacement})=(4/10)(3/9)=2/15$, not $(4/10)^2$. For data 1,2,3,4,20, $\text{mean}=30/5=6$ and $\text{median}=3$. Adding 5 to each shifts mean and median by 5 but leaves their pairwise deviations and standard deviation unchanged.

Figure 10: Worked reasoning



Check the labelled values and conditions.

11. Conditions and common errors

Conditions and common errors

A condition changes the reference population. Group means need weights, and no-replacement draws need changing denominators.

12. Original application

Original application

Group A has 12 observations with mean 5; group B has eight with mean 10. Find the combined mean. For the data 1,2,3,4,20 find mean, median and range. Describe how adding 7 to every value changes the standard deviation.

13. Model and reasoning

Model and reasoning

The combined total is $12(5) + 8(10) = 140$ across twenty observations, so mean is seven. An unweighted average of five and ten would incorrectly ignore group sizes. In the second data set, sum is thirty, mean six, median three and range nineteen. Adding seven shifts all values and their mean together; every deviation from the mean is unchanged, so standard deviation is unchanged.

14. Try independently

Try independently

Of 40 students, 24 study art, 18 study music and 10 study both. Find the probability of music given art and the probability of at least one subject. Are the events independent? Then explain what happens to standard deviation if all measurements are multiplied by -3.

15. Reasoning and check

Reasoning and check

Conditioning on art gives $P(M | A) = 10/24 = 5/12$. The union count is $24 + 18 - 10 = 32$, so union probability is $4/5$. Independence would require $P(A \cap M) = P(A)P(M)$, but $10/40 = 1/4$ differs from $(24/40)(18/40) = 27/100$. The events are not independent. Scaling every deviation by -3 scales its square by nine and the square root by three, so standard deviation triples. It cannot become negative.

Check yourself

Reconstruct the method without looking back

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