

Algebra: domains, systems, inequalities and functions

Original classroom teaching; not an official form or score predictor

1. Scope and task

Scope and task

Original classroom practice within the reviewed shorter-test task types. This does not simulate an adaptive test or predict a scaled or writing score. Earlier official public forms remain exposed practice; their version and writing-source holds still apply.

2. Scope and task

Scope and task

Solve equations while preserving denominator and radical restrictions Analyse inequalities and simultaneous conditions without losing endpoints Evaluate functions and translate verbal relationships into algebra

3. Scope and task

Scope and task

domain restriction : A condition identifying inputs on which the original expression is defined.

4. Scope and task

Scope and task

simultaneous solution : Values satisfying every relation in a system at the same time.

5. Read the evidence and choose a method

Read the evidence and choose a method

Write restrictions before manipulating an equation. A denominator must be nonzero; a real even root needs a nonnegative argument. Cancelling factors preserves the expression only on its original domain. Squaring can introduce candidates because a square loses sign information; verify every candidate in the unsquared relation.

6. Read the evidence and choose a method

Read the evidence and choose a method

Solving a system requires the same ordered pair to satisfy all relations. Substitution or elimination can expose one, no or infinitely many linear solutions. For an inequality, multiplication or division by a negative reverses direction. For a factored quadratic inequality, order roots and test interval signs, including endpoints only when equality is allowed.

7. Read the evidence and choose a method

Read the evidence and choose a method

A function assigns one output to each permitted input. Evaluate the inner function first in a composition, preserving domain restrictions. Read a slope as change in output per input unit and an intercept as the output at input zero only when that input is meaningful. A verbal model should name variables and keep quantities in compatible units.

8. Read the evidence and choose a method

Read the evidence and choose a method

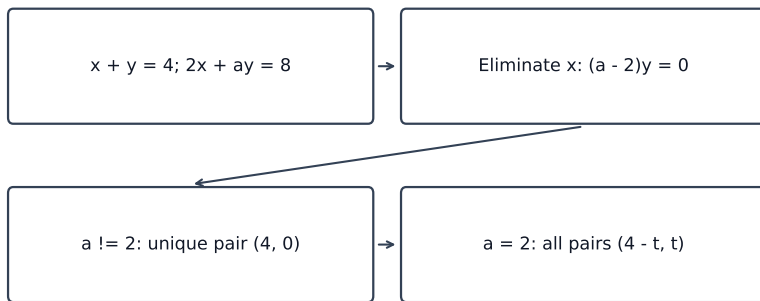
For Quantitative Comparison, preserve all allowed values. A variable with $x^2=4$ could be -2 or 2 unless constrained. One counterexample can reject a universal relationship; several favourable substitutions cannot prove it for every real value. Algebraic structure can establish a relationship without exhaustive numerical sampling.

9. Worked reasoning

Worked reasoning

For $\sqrt{x+2}=x$, $x \geq 0$. Squaring gives $x^2 - x - 2 = 0$, candidates 2 and -1 . Only 2 satisfies the original. For $(x-1)(x-4) \leq 0$, an upward quadratic is nonpositive on $1 \leq x \leq 4$, including roots. In the system $x+y=10$, $x-y=2$, adding gives $2x=12$; $x=6$ and $y=4$. For $f(t)=2t+1$ and $g(x)=x^2$, $f(g(3))=2 \cdot 9+1=19$, applying g first.

Figure 10: Worked reasoning



Check the labelled values and conditions.

11. Conditions and common errors

Conditions and common errors

A transformed equation supplies candidates, not automatic original solutions. Negative inequality multipliers and strict endpoints need explicit checks.

12. Original application

Original application

Solve $(x^2 - 4)/(x - 2) = 5$ on its original real domain. Solve $(x + 1)(x - 3) \leq 0$ and find $f(g(2))$ for $f(t) = t^2$, $g(x) = x - 3$.

13. Model and reasoning

Model and reasoning

The rational equation excludes $x=2$. For allowed x it reduces to $x+2=5$, giving $x=3$, which substitutes correctly. The quadratic is nonpositive between its roots, including them: $-1 \leq x \leq 3$. The composition applies g first: $g(2) = -1$ and $f(g(2)) = 1$. Cancelling a factor does not restore excluded inputs.

14. Try independently

Try independently

For real a , classify the system $x + y = 4$, $2x + ay = 8$ as having one, none or infinitely many solutions. Compare with the second equation $2x + ay = 9$. Give solutions in every consistent case.

15. Reasoning and check

Reasoning and check

Subtract twice the first equation. In the first system $(a - 2)y = 0$: for a not equal to two, $y=0, x=4$, uniquely. For $a=2$, both equations are the same relation, giving every pair $(4 - t, t)$. There is no inconsistent case in this first system. With right side nine, $(a - 2)y = 1$. For a not equal to two, $y = 1/(a - 2)$ and $x = 4 - 1/(a - 2)$, uniquely. For $a=2$ it demands $0=1$, so no solution. A zero coefficient row means inconsistency only when the right side is nonzero.

Check yourself

Reconstruct the method without looking back

For real a , classify the system $x + y = 4$, $2x + ay = 8$ as having one, none or infinitely many solutions. Compare with the second equation $2x + ay = 9$. Give solutions in every consistent case.