

Arithmetic: divisibility, remainders and multiplicative change

Original classroom teaching; not an official form or score predictor

1. Scope and task

Scope and task

Original classroom practice within the reviewed shorter-test task types. This does not simulate an adaptive test or predict a scaled or writing score. Earlier official public forms remain exposed practice; their version and writing-source holds still apply.

2. Scope and task

Scope and task

Use prime factorisation for divisibility, greatest common factors and least common multiples Interpret remainders with the original divisor Combine ratios and percentage changes with the correct reference total

3. Scope and task

Scope and task

least common multiple
given positive integer.

: The smallest positive integer divisible by each

4. Scope and task

Scope and task

remainder : The nonnegative amount left after division by a positive integer, smaller than that divisor.

5. Read the evidence and choose a method

Read the evidence and choose a method

An integer divisible by d can be written dk ; a remainder r gives $dk+r$ with $0 \leq r < d$ for a positive divisor. Prime factorisation identifies divisibility and common multiples. The greatest common factor takes shared prime powers at their smaller exponents; the least common multiple takes every needed prime power at its larger exponent.

6. Read the evidence and choose a method

Read the evidence and choose a method

Keep odd/even and positive/negative properties separate. A product of two odd integers is odd; a product with an even integer is even. A statement about integers need not hold for real numbers. For quantitative comparison, exploit a remainder form to expose possible cases rather than assume the smallest positive example is the only value.

7. Read the evidence and choose a method

Read the evidence and choose a method

A ratio $a:b$ allocates a whole into $a+b$ equal parts when the groups exhaust it. A percentage change multiplies the original value by $1+r$ or $1-r$ with r expressed as a decimal. Successive changes multiply factors. A part-of-a-part calculation changes the reference group at each stage; label the denominator before multiplying.

8. Read the evidence and choose a method

Read the evidence and choose a method

Estimate magnitude and check integer restrictions after solving. Fractions, roots and negative exponents follow their ordinary domain restrictions; even roots represent the nonnegative principal value. Do not cancel a term in a sum as if it were a factor. Exact arithmetic can be faster and safer than decimal calculator entries.

9. Worked reasoning

Worked reasoning

Known: $18 = 2 \cdot 3^2$ and $24 = 2^3 \cdot 3$. Shared smaller powers give GCF six; needed larger powers give LCM seventy-two. If $n = 5k + 2$, then $2n = 5(2k) + 4$, so the remainder on division by five is four.

10. Worked reasoning

Worked reasoning

For a model price of 100 CNY decreased then increased by 20%, multiply the two changes: $P_{\text{final}} = P_{\text{initial}}(1 - 0.20)(1 + 0.20)$.

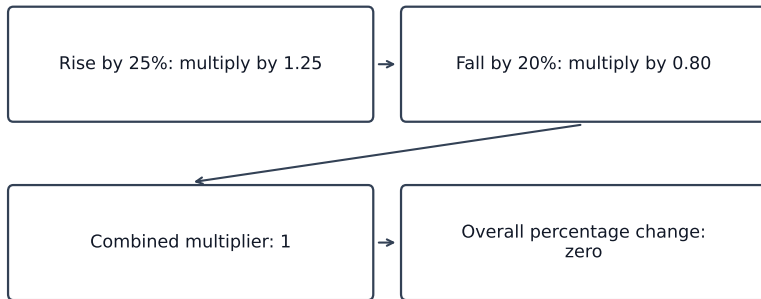
$P_{\text{final}} = (100 \text{ CNY})(0.8)(1.2) = 96 \text{ CNY}$. The price is not restored because the second percentage uses a different base.

11. Worked reasoning

Worked reasoning

Dividing 250 in the ratio 2:3 gives 100 and 150: each of five parts is fifty.

Figure 12: Worked reasoning



Check the labelled values and conditions.

13. Conditions and common errors

Conditions and common errors

Equal percentage increases and decreases use different bases. A remainder must be reported relative to the named divisor.

14. Original application

Original application

Find the greatest common factor and least common multiple of 36 and 60. A price rises 25% then falls 20%. Find its overall percentage change.

15. Model and reasoning

Model and reasoning

$36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$. Shared smaller powers give GCF=12; larger required powers give LCM=180. Their product $12 \cdot 180 = 36 \cdot 60$ checks the result. The price multiplier is $1.25 \cdot 0.80 = 1$, so the overall change is zero percent. The two changes use different reference prices; adding +25 and -20 would give a wrong five-percent increase.

16. Try independently

Try independently

An integer n has remainder 5 on division by 8. Find the remainder of $3n + 7$ on division by 8, including for negative n . A value rises 10% in one year and 20% the next. What single percentage reduction restores its original value?

17. Reasoning and check

Reasoning and check

Write $n = 8k + 5$ for integer k . Then $3n + 7 = 24k + 22 = 8(3k + 2) + 6$, so the remainder is six for all integer k . The final value is 1.32 times the original. Restoring it multiplies by $1/1.32 = 25/33$, a reduction of $1 - 25/33 = 8/33$, or about 24.24%. A 30% reduction does not reverse compounded growth. The remainder convention uses $0 \leq r < 8$ even when n is negative.

Check yourself

Reconstruct the method without looking back

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