

GAC004 Mathematics I: Fundamentals

GAC004-aligned fundamentals practice; the centre's brief sets the actual assessment and grading.

The discount that never came home

A price can fall by 20% and then rise by 20% — and still end up poorer than it started.

The method, every time

- Show the known data, say why the formula applies, write the symbols, then substitute.
- State the unit and interpret the result in one sentence.
- Read the command word: solve finds values, simplify rewrites, justify gives a reason.
- A reader must be able to check how the result follows.

The rules of the game

- Your centre's current assessment brief sets the tasks, weights, tools and deadlines.
- An in-class test, projects, an examination and coursework can each carry different instructions.
- A terminology logbook records the English term, your own meaning, and a small example.
- The practice sheets are original GAC004-aligned material, not official papers.

The number zoo

Term	What it is
Integer	A whole number: negative, zero or positive
Rational number	A fraction of two integers with a nonzero denominator
Irrational number	Cannot be such a fraction: π and $\sqrt{2}$ are the standard exhibits
Prime number	A positive integer above 1 with exactly two positive factors: itself and 1

Use the evidence

One is not prime. It has only one positive factor, and a prime insists on exactly two.

Order of operations: no negotiating

- 1 **The expression** $3 + 4 \times 2^2$ — brackets first, then indices, then multiplication and division, then addition and subtraction.
- 2 **Indices first** $2^2 = 4$, so the expression becomes $3 + 4 \times 4$.
- 3 **Multiplication before addition** $4 \times 4 = 16$, giving $3 + 16$.
- 4 **The result** 19 — not 28, which is what a left-to-right reading would invent.

Decision

The order of operations fixes what the expression means: 19, every time.

$$3 + 4 \times 2^2 = 3 + 4 \times 4 = 3 + 16 = 19$$

Factors, multiples and percentage multipliers

- A factor divides exactly; a multiple is what you multiply to get. 6 is a factor of 24, 24 a multiple of 6.
- The highest common factor (HCF) and lowest common multiple (LCM) come from prime factors: $24 = 2^3 \times 3$ and $36 = 2^2 \times 3^2$ give 12 and 72.
- An increase of 15% multiplies by 1.15; a decrease multiplies by 0.85. Reversing a change divides.
- Significant figures state precision; standard form writes $a \times 10^n$ with the coefficient between 1 and 10.

Worked example: down 20%, up 20%



each multiplier uses the current amount

1 Known An invented price of 80 units falls by 20%, then rises by 20%. Each change uses the current price.

2 The decrease $P_1 = 80(1 - 20/100) = 64$.

3 The increase $P_2 = 64(1 + 20/100) = 76.8$.

Decision

Same percentages, different bases: the decrease was 16, the rise only 12.8, so 80 becomes 76.8.

Your turn: undoing the rise

Your turn

A price rises by 25%. Which multiplier returns it to the original price?

- A. $\times 0.75$ — take 25% back off
- B. $\times 0.8$ — divide by 1.25
- C. $\times 1.25$ — the same multiplier again
- D. $\times 1.00$ — rises and falls always cancel

The anatomy of an expression

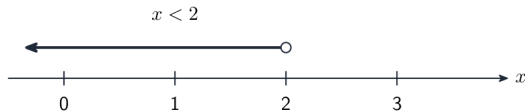
Word	Meaning
Expression	The whole thing: $5x^2 - 3x + 7$
Terms	The pieces: $5x^2$, $-3x$ and 7
Coefficient	The number in front: 5 is the coefficient of x^2
Constant	The term with no letter: 7
Equation	Two expressions declared equal, solved for a value
Identity	True for every value in its domain
Inequality	A comparison with $<$, at most, $>$ or at least

Expand and factorise: opposite directions

- To expand is to remove brackets; to factorise is to put them back.
- $(x + 3)(x - 5)$ expands to $x^2 - 2x - 15$, term by term.
- A linear equation is solved by doing the same operation to both sides until the letter stands alone.
- A quadratic equation $ax^2 + bx + c = 0$ factorises or takes the formula; simultaneous equations fall to substitution or elimination.

$$(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$$

Worked example: the sign that flipped



- 1 Known** Solve $7 - 3x > 1$. The same operation applies to both sides.
- 2 Subtract 7** $-3x > -6$.
- 3 Divide by -3 — and reverse** $x < (-6)/(-3) = 2$. Dividing by a negative reverses the inequality.

Decision

The open circle excludes 2; a check with $x = 1$ gives $4 > 1$.

Your turn: which direction

Your turn

Solve $-2x > 6$.

A. $x > -3$

B. $x < -3$

C. $x > 3$

D. $x < 3$

A line, measured

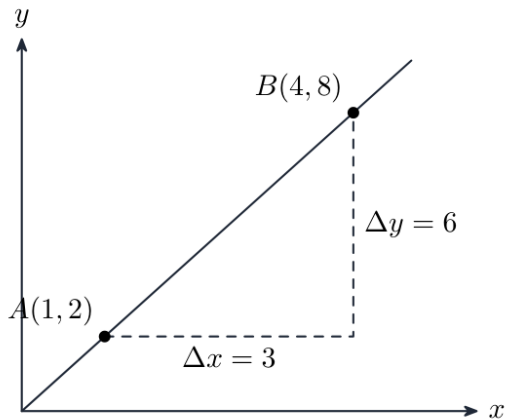
- A nonvertical line is $y = mx + c$: gradient m , y -intercept c .
- The gradient is rise over run: $(y_2 - y_1) / (x_2 - x_1)$.
- Parallel lines share a gradient; perpendicular gradients multiply to -1 .
- A vertical line has undefined gradient and pairs perpendicularly with a horizontal one.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The coordinate toolkit

Tool	What it gives you
Midpoint	The average of the endpoints, coordinate by coordinate
Distance	Pythagoras between the points: root of the squared differences
Parabola	A quadratic graph: the vertex is its turning point
Roots	The x-values where the curve meets the axis: $y = 0$
Graphical solution	Solve graphically: read the intersection; only as accurate as the drawing

Worked example: the line through A and B



- Known: $A = (1, 2)$ and $B = (4, 8)$, a nonvertical line, so use the gradient.
- $m = (8 - 2) / (4 - 1) = 2$.
- Substitute A into $y = mx + c$: $c = 2 - 2(1) = 0$, so $y = 2x$.

Your turn: the perpendicular

Your turn

A line is perpendicular to $y = 2x + 5$. What is its gradient?

- A. 2
- B. -2
- C. $1/2$
- D. $-1/2$

The angle facts

Where	The angles add to
On a straight line	180 degrees
Around a point	360 degrees
Inside a triangle	180 degrees
Inside an n-sided polygon	$(n - 2) \times 180$ degrees

Use the evidence

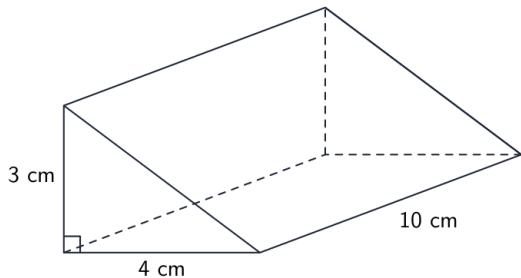
The polygon rule counts the triangles the polygon can be cut into: $n - 2$ of them.

Same shape, or same everything

- Congruent shapes are identical in size and shape; similar shapes share the shape with lengths in one ratio.
- Pythagoras' theorem: $a^2 + b^2 = c^2$, with c the hypotenuse.
- A cylinder has volume $\pi r^2 h$; a sphere has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.
- Scaling is not linear: double every length and area multiplies by 4, volume by 8.

$$a^2 + b^2 = c^2; \quad V_{\text{cyl}} = \pi r^2 h; \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

Worked example: the triangular prism



not to scale

1 Known Right-triangle cross-section, perpendicular sides 3 and 4 cm; prism length 10 cm. A constant cross-section means $V = \text{area} \times \text{length}$.

2 The cross-section
 $A = \frac{1}{2}bh = \frac{1}{2}(3)(4) = 6 \text{ cm}^2$.

3 The volume
 $V = A \times L = 6 \times 10 = 60 \text{ cm}^3$.

Decision

The sloping side is not the height; the end area is not the volume.

Your turn: the scaled model

Your turn

Every length of a model boat is multiplied by 3. Its volume is multiplied by what?

- A. 3
- B. 6
- C. 9
- D. 27

SOH CAH TOA: the three ratios

Ratio	Equals
sin of the angle	opposite over hypotenuse
cos of the angle	adjacent over hypotenuse
tan of the angle	opposite over adjacent

Use the evidence

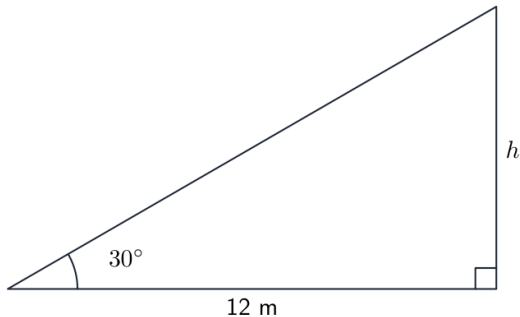
SOH CAH TOA: say it once and the three ratios stay labelled for life.

Beyond the right angle

- The sine rule pairs sides with the sines of opposite angles; the cosine rule generalises Pythagoras.
- Angles come in degrees or radians: π radians is 180 degrees, so check the calculator mode every time.
- The unit circle extends the ratios beyond 90 degrees: sine and cosine are periodic, repeating every 360 degrees.
- A bearing is clockwise from north with three figures; angles of elevation and depression are measured from the horizontal.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}; \quad a^2 = b^2 + c^2 - 2bc \cos A$$

Worked example: the height from 12 metres



not to scale

- Known: horizontal distance $d = 12$ m, angle of elevation 30 degrees.
- The height is opposite and the distance adjacent, so use tangent in degree mode.
- $h = 12 \tan 30$, about 6.93 m above the sight level, not automatically above the ground.

$$h = d \tan \theta = 12 \tan 30^\circ \approx 6.93 \text{ m}$$

Your turn: pick the ratio

Your turn

You know the hypotenuse and want the side opposite the angle. Which ratio?

- A. sine
- B. cosine
- C. tangent
- D. the cosine rule

Growth and decay, one formula apart

- An exponential function $y = a^x$ puts the variable in the exponent; each unit step multiplies by a .
- It grows when $a > 1$; exponential decay uses a between 0 and 1; the y-intercept is 1.
- The x-axis is a horizontal asymptote: the values stay positive.
- Compound interest is exponential: $A = P(1 + r)^n$ after n periods.

$$y = a^x \ (a > 0, \ a \neq 1); \quad A = P(1 + r)^n$$

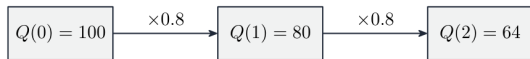
Logarithms ask the reverse question

Law	Statement
The definition	$\log_a y = x$ means exactly $a^x = y$
The product law	$\log(mn) = \log m + \log n$
The quotient law	$\log(m/n) = \log m - \log n$
The power law	$\log(m^k) = k \log m$
Natural logarithms	ln uses base e, about 2.718

Use the evidence

The argument must be positive and the base positive and not 1; the graph passes through (1, 0) with asymptote $x = 0$.

Worked example: decay multiplies, never subtracts



equal time periods, changing absolute reductions

- Known: a quantity starts at 100 units and loses 20% per whole period, so the retained proportion is 0.8.
- $Q(2) = 100 \times (0.8)^2 = 64$.
- The first loss is 20 units, the second 16: a fixed subtraction of 20 would be a different model.

$$Q(n) = Q_0(1 - r)^n; \quad Q(2) = 100(0.8)^2 = 64$$

Your turn: free the exponent

Your turn

Solve $2^x = 32$.

- A. $x = 5$
- B. $x = 16$
- C. $x = 6$
- D. $x = 4$

Check yourself

- A price falls 20% then rises 20%: why is it not back where it started?
- In $-2x > 6$, what happens to the sign, and why?
- Doubling every length of a solid: what happens to its area and to its volume?
- Which log law frees an unknown sitting in an exponent?