

GAC004 Extra Practice: Percentages, Inequalities and Similar Shapes

Four original worked questions with full methods and checks, for sheets 1.1, 1.2 and 1.4

What this pack contains

Question	Sheet	What it practises
Q1	1.1	Two price changes in a row
Q2	1.1	Undo one increase
Q3	1.2	Divide by a negative number
Q4	1.4	From lengths to areas

Try each question before you open its answer. A clear, stated method shows your reasoning better than a long report.

How to answer, step by step

Write what is known, with units.



Say why the rule applies.



Write the rule in symbols first.



Put in the numbers, one stage per line.

The check at the end is part of the answer, not extra.

Q1: try it first

Your turn

A price of 200 units rises by 10%, then falls by 10%.

Find the final price.

Explain why the final price is not 200 units.

Write the known numbers and your method before the next slide.

Q1: the rise, in symbols then numbers

1 **Known** $P_0 = 200$ units; rise $r = 10\%$; fall $r = 10\%$.

2 **Reason** Each **percentage** uses the current price, so apply a **multiplier** at each stage.

The increase **equation** , in symbols:

$$P_{new} = P_{old}(1 + r/100)$$

$$P_1 = P_0(1 + r/100) = 200(1 + 10/100) = 220$$

Q1: the fall, and the final price

The decrease, in symbols:

$$P_{new} = P_{old}(1 - r/100)$$

$$P_2 = P_1(1 - r/100) = 220(1 - 10/100) = 198$$



Each multiplier acts on the price at that stage.

Answer

The final price is **198** units.

Q1: why the price does not return to 200

The two multipliers form one combined multiplier:

$$\frac{P_2}{P_0} = (1 + r/100)(1 - r/100)$$

$$\frac{P_2}{P_0} = (1 + r/100)(1 - r/100) = (1 + 10/100)(1 - 10/100) = 0.99$$

What 0.99 means

The final price keeps 99% of the starting price.

The overall change

The overall loss is 1%, not zero.

Q1: the two changes use different amounts

The rise added

$$\Delta_{rise} = P_1 - P_0 = 220 - 200 = 20$$

That is 10% of 200.

The fall removed

$$\Delta_{fall} = P_1 - P_2 = 220 - 198 = 22$$

That is 10% of 220.

10% of 220 is larger than 10% of 200: the fall removes more than the rise added.

Q1: check the overall loss two ways

Check: the overall loss is 1% of the starting price.

$$1 - 0.99 = 0.01$$

$$P_0 \times 0.01 = 200 \times 0.01 = 2$$

$$P_0 - P_2 = 200 - 198 = 2$$

Both give 2 units, so the two checks agree.

Q2: try it first

Your turn

After a 20% increase, a figure is 84.

Find the original figure.

Be careful: the increase was not calculated on 84.

Q2: rearrange the increase equation

- 1 Known** New figure $N = 84$; increase $r = 20\%$.
- 2 Reason** The increase multiplied the original by $1 + r/100$. Undo it by dividing by this multiplier. Never subtract the same percentage back.

The increase equation, in symbols:

$$N = P_0(1 + r/100)$$

Rearrange for the original figure:

$$P_0 = \frac{N}{1 + r/100}$$

Q2: substitute, then divide

Substitute:

$$P_0 = \frac{N}{1 + r/100} = \frac{84}{1 + 20/100}$$

Work out the multiplier:

$$1 + \frac{r}{100} = 1 + \frac{20}{100} = 1.2$$

Divide:

$$P_0 = \frac{N}{1 + r/100} = \frac{84}{1.2} = 70$$

Answer

Q2: reverse check, and the trap

Reverse check: run the increase forward again

$$N = P_0(1 + r/100) = 70(1 + 20/100) = 84$$

The original 70 returns the stated 84, so the reversal is correct. ✓

The trap: subtracting 20% of the new figure

$$84 - 84 \times 0.20 = 67.2$$

That is wrong. The increase was calculated on 70, not on 84. ✗

To undo a percentage change, divide by the multiplier.

Q3: try it first

Your turn

Solve $9 - 2x \leq 3$.

Show your solution on a **number line** .

Solve it like an equation, but watch the last operation.

Q3: subtract 9 from both sides

1 Known The **inequality** $9 - 2x \leq 3$.

2 Rule Dividing both sides by a negative number **reverses** the sign.

Subtract 9 from both sides:

$$9 - 2x \leq 3$$

$$-2x \leq 3 - 9$$

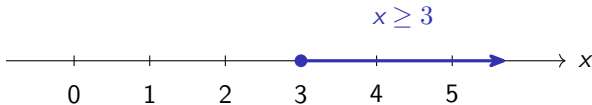
$$-2x \leq -6$$

Q3: the solution on the number line

Divide both sides by -2 , and reverse the sign:

$$x \geq \frac{-6}{-2} = 3$$

The solution is $x \geq 3$.



Q3: the boundary and the closed circle

The **boundary** is included. At $x = 3$ the two sides are equal:

$$9 - 2x = 9 - 2(3) = 9 - 6 = 3$$

Why the circle is closed

$x = 3$ satisfies the inequality, so 3 belongs to the solution.

Why the shading goes right

Values greater than 3 satisfy it. Values below 3 fail.

Mark a **closed circle** at 3, then shade to the right.

Q3: test one value on each side

Inside the solution: $x = 4$

$$9 - 2x = 9 - 2(4) = 9 - 8 = 1$$

$1 \leq 3$  So $x = 4$ satisfies the inequality.

Outside the solution: $x = 2$

$$9 - 2x = 9 - 2(2) = 9 - 4 = 5$$

$5 > 3$  So $x = 2$ fails the inequality.

Both checks agree with $x \geq 3$.

Q4: try it first

Your turn

Two **similar** models have a **length ratio** of 3 : 5.

The smaller model has **surface area** 36 cm^2 .

Find the larger surface area.

Hint: areas do not scale by the same factor as lengths.

Q4: from a length ratio to an area ratio

- 1 Known** Length ratio 3 : 5; smaller surface area $A_s = 36 \text{ cm}^2$.
- 2 Reason** Let k be the **scale factor** from a smaller length L_s to the matching larger length L_L . Areas scale as the **square** of k , because each area is a product of two lengths.

The general relation between the two areas:

$$\frac{A_L}{A_s} = k^2$$

The length ratio gives the scale factor:

$$k = \frac{L_L}{L_s} = \frac{5}{3}$$

Q4: the area ratio, then the larger area

The area ratio:

$$\frac{A_L}{A_s} = k^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

Rearrange for the larger area:

$$A_L = A_s k^2$$

Substitute:

$$A_L = A_s k^2 = 36 \times \frac{25}{9} = 100$$

The larger surface area is **100** cm².

Q4: ratio check and the scaling limit

Ratio check

$$\frac{A_L}{A_s} = \frac{100}{36} = \frac{25}{9}$$

$$\frac{A_L}{A_s} = k^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

The area ratio 25 : 9 is the square of the length ratio 5 : 3.

The scaling limit

Only areas scale by $\frac{25}{9}$.

A larger length is $k = \frac{5}{3}$ times the smaller length.

Match the scaling factor to the quantity you are asked for.

Four traps to avoid

The trap	What to do instead
Add and subtract 10% as fixed amounts	Apply a multiplier to the current price
Subtract 20% of the new figure	Divide by the increase multiplier
Keep the sign when dividing by a negative	Reverse the sign
Scale areas by the length factor	Square the scale factor for areas

Each trap appeared in this pack. Check your own answers for them.

Check yourself

- 1 Symbols** Can you write each rule in symbols before the numbers?
- 2 Checks** Can you run the reverse check and the boundary check yourself on a fresh copy?
- 3 Next practice** Record the step where your method differed, then try a new question on sheets 1.1, 1.2 and 1.4.

A stated method with a check shows your reasoning clearly; a bare number does not.