GAC024 Discrete Mathematics

GAC Mathematics

What this module is, and how it is marked

GAC024 is discrete mathematics: the mathematics of things you can count and of the logic computers run on. Five units cover sets, counting systems, binary, algorithms, and graphs.

It pairs naturally with the computing modules, and universities that credit GAC016 often also recognise this one for a computer-science foundation course. Assessment follows the usual pattern: a test, an examination, and coursework.

- △ Discrete mathematics has few formulas and much **reasoning**. A proof or a traced algorithm carries the marks, and an unexplained answer earns almost nothing.

Sets, relations and functions

- A **set** 集合 is a collection of distinct objects. Order and repetition do not matter.
- **Union** 并集 $A \cup B$ is everything in either; **intersection** 交集 $A \cap B$ is what is in both; the **set complement** 补集 is everything outside.
- A **subset** 子集 has all its elements inside another set.
- A **relation** 关系 pairs elements of two sets. A **function** 函数 is a relation where each input has exactly one output.
- A **Venn diagram** 韦恩图 turns a set problem into a picture, and drawing one is usually faster than reasoning about it in words.

Worked example. In a class of 30, 18 study French and 15 study German; 7 study both. How many study neither?

$$|F \cup G| = 18 + 15 - 7 = 26 \quad \Rightarrow \quad 30 - 26 = 4$$

Subtracting the overlap once is the **inclusion-exclusion principle** 容斥原理. Adding 18 and 15 without it counts the seven twice, which is the standard error.

Counting systems

- A **number base** 进制 says how many digits it uses. **Decimal** 十进制 uses ten, **binary** 二进制 two, **hexadecimal** 十六进制 sixteen.

- Every digit's value is its **place value** 位值: in binary the places are 1, 2, 4, 8, 16 and so on.
- Hexadecimal is shorthand for binary: one hex digit is exactly four bits, which is why memory addresses are written in it.

Worked example. Convert 1101 in binary to decimal.

$$8 + 4 + 0 + 1 = 13$$

Write the place values above the digits before you add. Doing it in your head is where the off-by-one errors come from.

Binary applications

- **Binary arithmetic** 二进制运算 adds like decimal, carrying at 2 instead of at 10.
- A **bit** 位 is one binary digit; a **byte** 字节 is eight.
- **Boolean algebra** 布尔代数 works on true and false with AND, OR and NOT.
- A **truth table** 真值表 lists every input combination and its output, and it is the complete proof that two logic expressions are equivalent.
- **Logic gates** 逻辑门 are the physical form of those operations, and a **logic circuit** 逻辑电路 is what a processor is made of.

Algorithms

- An **algorithm** 算法 is a finite sequence of unambiguous steps that terminates.
- A **flowchart** 流程图 draws it: a decision is a diamond, a process a rectangle.
- **Pseudocode** 伪代码 writes it in structured English, which is what an exam usually asks for.
- **Tracing** 追踪 an algorithm — a table with one column per variable and one row per step — is the marked technique, and it is how you find a bug without running anything.
- **Efficiency** 效率 matters: a linear search checks every item, a binary search halves the list each time, and on a million items that is the difference between a million steps and twenty.

Worked example. Trace a binary search for 7 in [1, 3, 5, 7, 9, 11].

Middle is 5, which is less than 7, so search the right half. Middle of [7, 9, 11] is 9, which is greater, so search the left. Middle of [7] is 7. Found, in three steps rather than four.

The table of steps is the answer. The word "found" is not.

Graphs and networks

- A **graph** 图 is a set of **vertices** 顶点 joined by **edges** 边. It models anything with connections: roads, friendships, dependencies.
- The **degree** 度 of a vertex is how many edges meet it.
- A **tree** 树 is a connected graph with no cycles, and it is the shape of a file system, an organisation chart and an HTML document.
- A **shortest path** 最短路径 problem asks for the cheapest route between two vertices, and it is what a navigation app solves every time you use it.