GAC010 Mathematics II: Probability, Statistics and Finance

GAC Mathematics

What this module is, and how it is marked

GAC010 is the Level II mathematics module, and it is the one that most resembles the mathematics a university degree in business, economics or the social sciences will actually use: sequences, money, probability and statistics.

Assessment follows the pattern of GAC004 — a **test** 测验 on the early units, one or two **projects** 项目 built in a spreadsheet, an **examination** 考试 over all eight units, and **coursework** 平时作业 including the terminology logbook. Several universities publish credit for this module, so treat it as a first-year course rather than as revision.

- △ The financial units reward **method shown**. A single number with no working earns almost nothing, because the marker cannot see which formula you chose.
- Statistics questions ask you to **interpret**, not only to compute. "The mean is 42" is half an answer; "the mean is 42, but the two outliers pull it above the median" is the other half.

Sequences and series

A **sequence** 数列 is an ordered list of numbers; a **series** 级数 is their sum.

An **arithmetic sequence** 等差数列 adds a fixed **common difference** 公差 d each step. A **geometric sequence** 等比数列 multiplies by a fixed **common ratio** 公比 r.

$$u_n = a + (n-1)d \qquad \text{and} \qquad u_n = ar^{\,n-1}$$

- The sums are $S_n = \frac{n}{2}\,(2a + (n-1)d)$ and $S_n = \dfrac{a(1-r^n)}{1-r}$ for $r \neq 1$.
- A geometric series with $|r| < 1$ has a **sum to infinity** 无穷和 $S_\infty = \dfrac{a}{1-r}$.
- The distinction matters because money is geometric: interest multiplies, it does not add.

Worked example. A machine loses 12% of its value each year from 80,000 yuan. What is it worth after 5 years?

$$u_6 = 80\,000 \times 0.88^5 = 42\,200 \text{ yuan (3 s.f.)}$$

Note the exponent: after 5 years you have applied the ratio five times, so the term is the sixth. Off-by-one here is the most common error in the unit.

Finance: earning money and purchasing goods

This unit is the arithmetic of an ordinary life, in English.

- **Gross pay** 税前工资 is what you earn; **net pay** 税后工资 is what arrives after **deductions** 扣除项 such as tax and insurance.
- **Simple interest** 单利 is $I = Prt$ — interest on the original amount only.
- **Compound interest** is $A = P(1+r)^n$ — interest on the interest, which is why it is geometric.
- **Depreciation** 折旧 is compound interest with a negative rate, and it is how the machine above lost its value.
- A **discount** 折扣 of 20% multiplies by 0.8. Two successive discounts of 20% and 10% multiply by $0.8 \times 0.9 = 0.72$, which is a 28% reduction, not 30%.

Finance: investing money and loans

- The **present value** 现值 of a future amount is what it is worth today: $PV = \dfrac{FV}{(1+r)^n}$.
- An **annuity** 年金 is a fixed payment each period; a **loan repayment** 贷款还款 is an annuity paid to a lender.
- The **APR** 年利率 lets you compare two loans whose headline rates are quoted over different periods.
- △ Nominal and effective rates differ when interest compounds more than once a year. 12% a year compounded monthly is 12.68% effective, and a question that says "monthly" is testing exactly this.

Worked example. Which is cheaper: borrowing 10,000 yuan at 9% a year compounded annually for 3 years, or at 8.7% compounded monthly?

$$10\,000 \times 1.09^3 = 11\,950 \qquad 10\,000\left(1+\tfrac{0.087}{12}\right)^{36} = 12\,970$$

The lower headline rate costs more, because it compounds 36 times instead of 3. Comparing the headline numbers is the trap; comparing the totals is the method.

Probability: key concepts

Probability 概率 measures how likely an event is, on a scale from 0 to 1.

- For equally likely outcomes, $P(A) = \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.
- Events are **mutually exclusive** 互斥 when they cannot both happen: $P(A \text{ or } B) = P(A) + P(B)$.
- Events are **independent** 独立 when one does not affect the other: $P(A \text{ and } B) = P(A) \times P(B)$.
- The **complement** 对立事件 rule, $P(\text{not } A) = 1 - P(A)$, is often the fastest route to an answer containing "at least one".
- **Counting techniques** 计数方法 supply the denominator: a **permutation** 排列 counts ordered selections, a **combination** 组合 counts unordered ones.

Worked example. A bag holds 4 red and 6 blue balls. Two are drawn without replacement. What is the probability of at least one red?

Use the complement: $P(\text{no red}) = \dfrac{6}{10} \times \dfrac{5}{9} = \dfrac{1}{3}$, so $P(\text{at least one red}) = \dfrac{2}{3}$.

Without replacement, the second fraction has both a smaller numerator and a smaller denominator. Forgetting to change the denominator is the standard error.

Statistics: collecting and displaying data

- A **population** 总体 is everyone you want to describe; a **sample** 样本 is who you actually measure.
- A sample is only useful if it is **representative** 有代表性的. A survey of your own class does not describe the school.
- **Qualitative data** 定性数据 are categories; **quantitative data** 定量数据 are numbers, either **discrete** 离散 (counted) or **continuous** 连续 (measured).
- **Bias** 偏差 enters through who you ask, who answers, and how the question is worded, and no amount of arithmetic afterwards removes it.

Statistics: measures of central location

- The **mean** 平均数 is the total divided by the count. It uses every value, and every outlier.
- The **median** 中位数 is the middle value in order. It ignores how extreme the extremes are.
- The **mode** 众数 is the most frequent value, and it is the only average that works for categories.

- △ Choose by the shape of the data. For income, the median describes a typical person and the mean does not, because a few very large values drag it upward.

Statistics: measures of variability

- The **range** 全距 is the largest minus the smallest — one number, easily destroyed by an outlier.
- The **interquartile range (IQR)** 四分位距 is the middle 50%, and it is what a **box plot** 箱线图 draws.
- The **standard deviation** 标准差 measures the typical distance from the mean: $\sigma = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n}}$.
- A larger standard deviation means a wider spread. Two data sets can share a mean and describe completely different situations.

Distributions: the normal distribution

- The **normal distribution** 正态分布 is the symmetric bell curve, described entirely by its mean and its standard deviation.
- The empirical rule: about 68% of values lie within one standard deviation of the mean, 95% within two, and 99.7% within three.
- A **z-score** 标准分 says how many standard deviations a value sits from the mean: $z = \dfrac{x - \mu}{\sigma}$, which is what makes two different scales comparable.

Worked example. A test has mean 62 and standard deviation 8. What proportion of students score above 78?

$$z = \frac{78 - 62}{8} = 2$$

Two standard deviations above the mean. By the empirical rule 95% lie within two either side, so 2.5% lie above — about 1 student in 40.