

Notes

Pearson Edexcel · International GCSE · Mathematics A

4MA1: course teaching notes

Version: Issue 2, November 2017; first assessment June 2018; linear Mathematics A

These are original course-owned teaching notes. Objective-level exceptions are listed in the review, and lessons are tier labelled.

1 · Exact arithmetic and estimation · Foundation

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** 质因数: A prime number that divides the integer exactly.

Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

$$72 = 2^3 \times 3^2, \quad 90 = 2 \times 3^2 \times 5, \quad \text{HCF} = 18$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

Shared smallest powers: $2 \times 3^2 = 18$

Test a tempting shortcut

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- 4MA1 · Foundation · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

1 · Exact arithmetic and estimation · Higher

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** 质因数: A prime number that divides the integer exactly.

Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

Shared smallest powers: $2 \times 3^2 = 18$

Test a tempting shortcut

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

1 · Integer indices and standard form · Foundation

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
 - This lesson studies **index** 指数: The power to which a base is raised.
-

Choose the mathematical structure

- Use integer powers, square and cube roots and standard form with $1 \leq a < 10$. In multiplying powers with the same base, add indices; in division, subtract them.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$a^m a^n = a^{m+n}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $2^3 \times 2^4 = 2^7 = 128$. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal.

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Test a tempting shortcut

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For every positive a , $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- This Foundation/Core lesson excludes fractional powers and surd rationalisation. Estimate a result before using a calculator and retain the required precision.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

1 · Indices, surds and standard form · Higher

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
 - This lesson studies **index** 指数: The power to which a base is raised.
-

Choose the mathematical structure

- For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{(3/4)} = (16^{(1/4)})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Test a tempting shortcut

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For every positive a , $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

1 · Percentages, ratio and proportional reasoning · Foundation

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
 - This lesson studies **multiplier** 乘数: A factor that performs a percentage change in one multiplication.
-

Choose the mathematical structure

- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

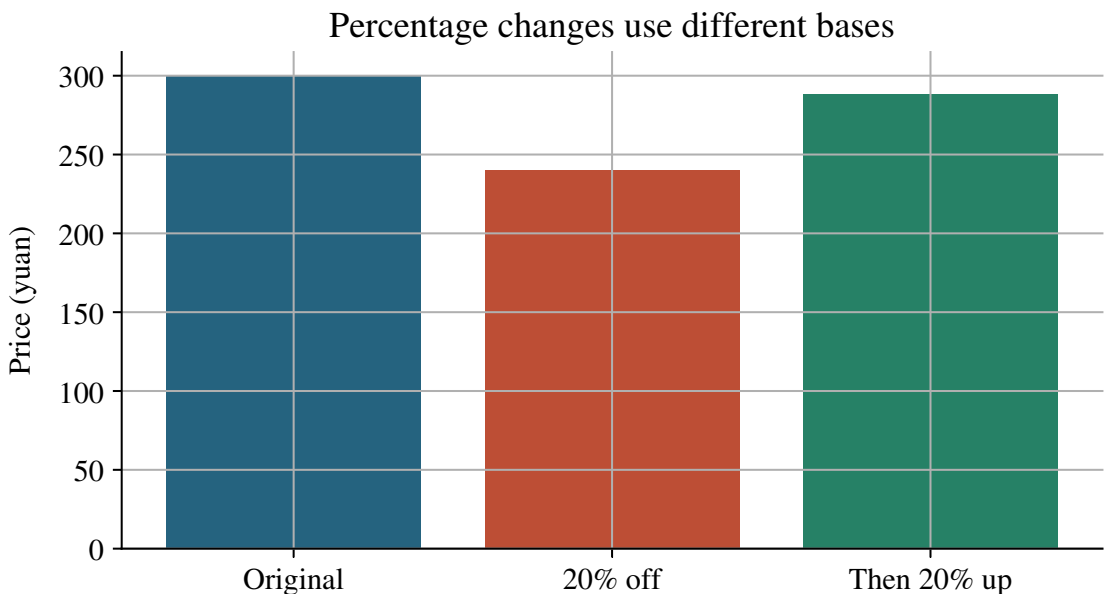
Work through a checked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



Test a tempting shortcut

- A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use percentage multipliers and divide a total into ratio parts. This Foundation/Core lesson uses linear proportional contexts, not the advanced regression methods.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

1 · Percentages, ratio and proportional reasoning · Higher

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
 - This lesson studies **multiplier** 乘数: A factor that performs a percentage change in one multiplication.
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Choose the mathematical structure

- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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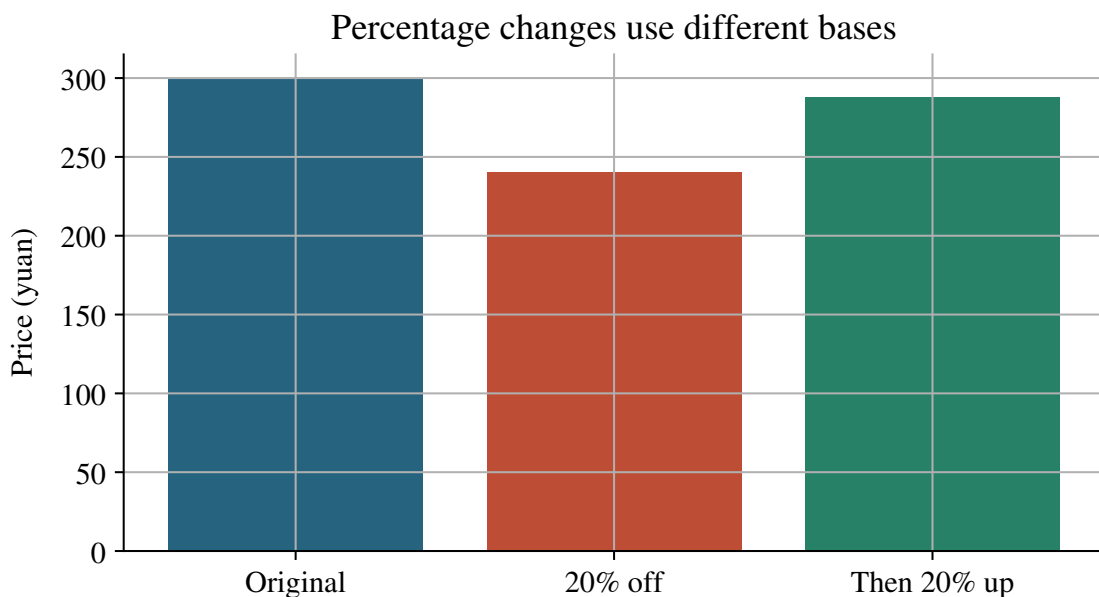
Work through a checked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

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Test a tempting shortcut

- A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

1 · Accuracy, bounds and compound measures · Higher

Is the printed measurement exact?

- A rectangular panel is labelled 8.0 cm by 5.0 cm, each to the nearest 0.1 cm. Its true area is not fixed at 40 cm².

- This lesson studies **lower bound** 下界: The smallest possible value consistent with a stated rounding rule.

Choose the mathematical structure

- A value rounded to the nearest unit u lies from stated value $-u/2$ up to, but usually not including, stated value $+u/2$. For positive quantities, combine extremes according to the operation.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

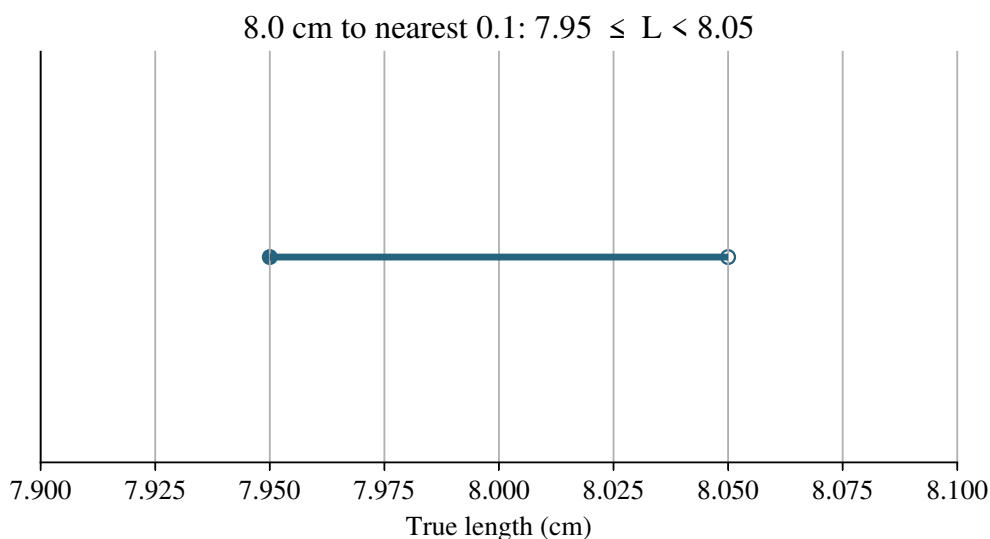
Work through a checked case

$$A = LW, \quad v = \frac{d}{t}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The lengths satisfy $7.95 \leq L < 8.05$ and $4.95 \leq W < 5.05$. Since $A = LW$, $39.3525 \leq A < 40.6525$. For speed d/t , the largest speed uses the largest distance and smallest positive time.



Test a tempting shortcut

- An upper bound is not automatically achieved. Dividing upper distance by upper time does not give the largest speed. Keep enough digits in intermediate calculations.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The greatest value of positive d/t uses the greatest d and greatest t . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Distinguish measurement uncertainty from arithmetic rounding. A sensible reported precision cannot be finer than the measurements justify.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 1. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The smallest possible value consistent with a stated rounding rule. Choose the relationship, show the method, check its assumptions and interpret the result.

2 · Equations, identities and rearrangement · Foundation

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
 - This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.
-

Choose the mathematical structure

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs. Preserve equality by applying the same operation to both sides. State restrictions before dividing by a variable.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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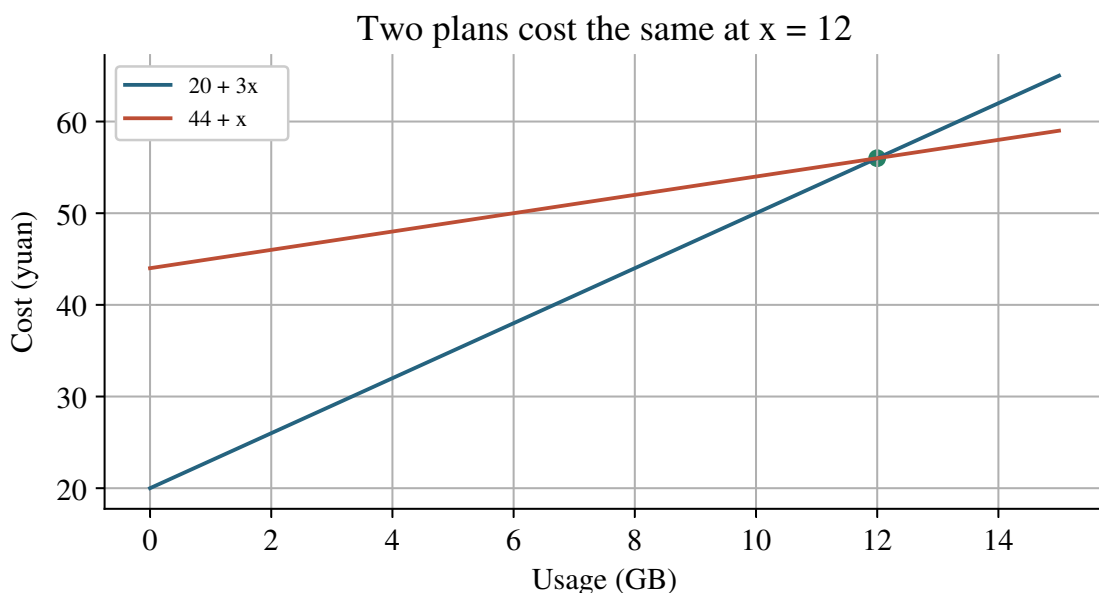
Work through a checked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{(A/\pi)}$, because r is a length.



Test a tempting shortcut

- Cancelling a term is not the same as cancelling a factor. In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$. Check a rearrangement by substitution.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Define the unknown and set up a linear equation. Check the answer by substitution. Restrict this Foundation/Core lesson to simple expressions and equations.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- 4MA1 · Foundation · 2. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An equality that holds for every allowed value of its variable. Choose the relationship, show the method, check its assumptions and interpret the result.

2 · Equations, identities and rearrangement · Higher

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
- This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.

Choose the mathematical structure

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs. Preserve equality by applying the same operation to both sides. State restrictions before dividing by a variable.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

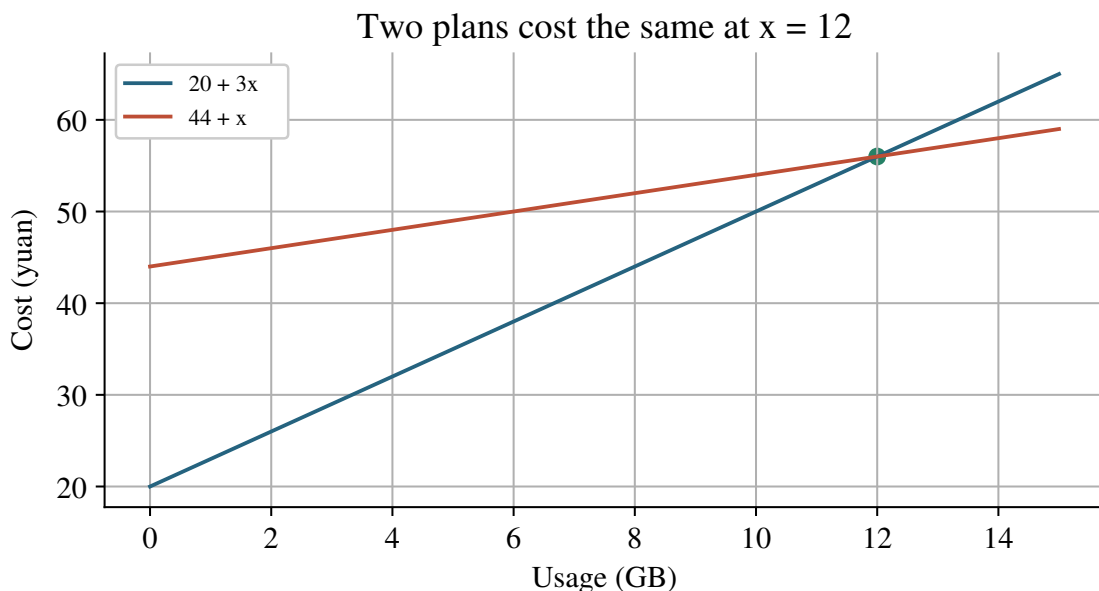
Work through a checked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{(A/\pi)}$, because r is a length.



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- Cancelling a term is not the same as cancelling a factor. In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$. Check a rearrangement by substitution.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Set up the equation from units and the meaning of the unknown. A negative or fractional solution may be algebraically correct but impossible for a count.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- 4MA1 · Higher · 2. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An equality that holds for every allowed value of its variable. Choose the relationship, show the method, check its assumptions and interpret the result.

2 · Factorising and solving simple quadratics · Foundation

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.

Choose the mathematical structure

- Expand brackets and factorise simple quadratics. Solve by setting each factor equal to zero, and use a graph to interpret the roots.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

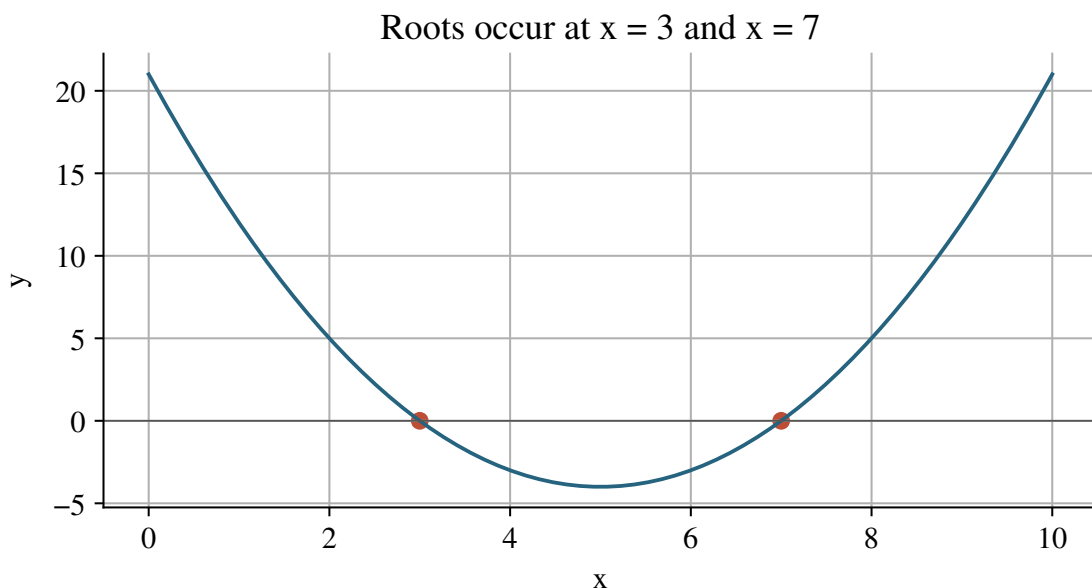
Work through a checked case

$$(x-3)(x-7)=0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$x^2 - 10x + 21 = (x-3)(x-7)$. Hence the equation $x^2 - 10x + 21 = 0$ has roots 3 and 7. Check each root by substitution and mark both intercepts on the graph.



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- This Foundation/Core lesson uses factorisation and graphical roots; the discriminant, quadratic formula and quadratic inequalities are reserved for the advanced tier.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 2. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

2 · Quadratics and inequalities · Higher

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
 - This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
-

Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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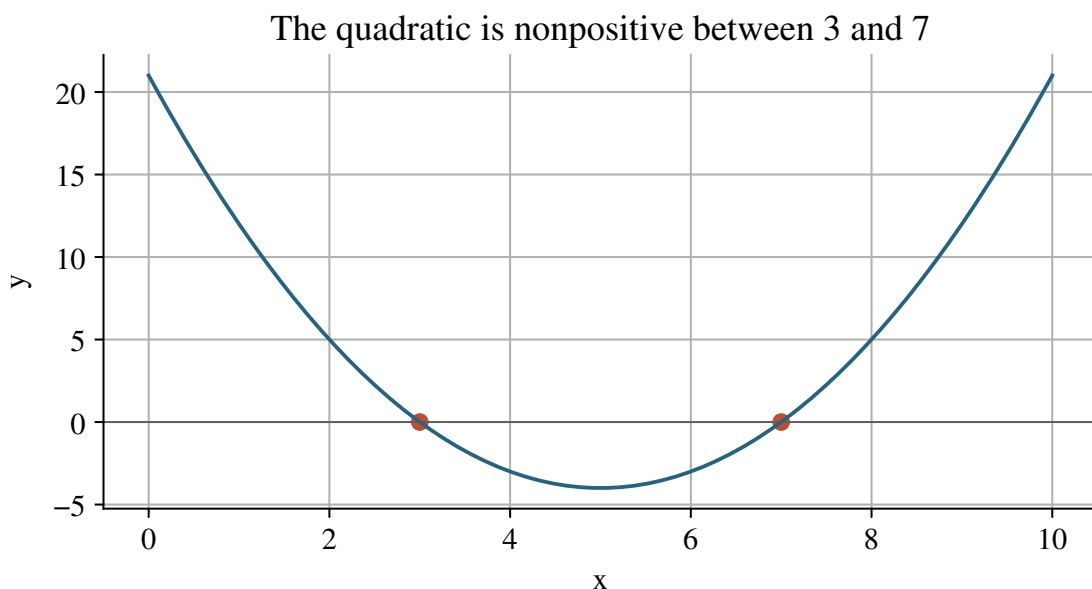
Work through a checked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 2. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

2 · Simultaneous equations and feasible regions · Foundation

Can one equation determine two counts?

- Two ticket types raise a total amount. A single equation cannot identify both unknown counts.
 - This lesson studies **elimination** 消元法: Combining equations to remove one variable while retaining the same solutions.
-

Choose the mathematical structure

- For two linear equations, use elimination or substitution and check both equations. For a line and a quadratic, substitute the linear relation first; then solve the resulting quadratic. For inequalities, shade the region satisfying every condition.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

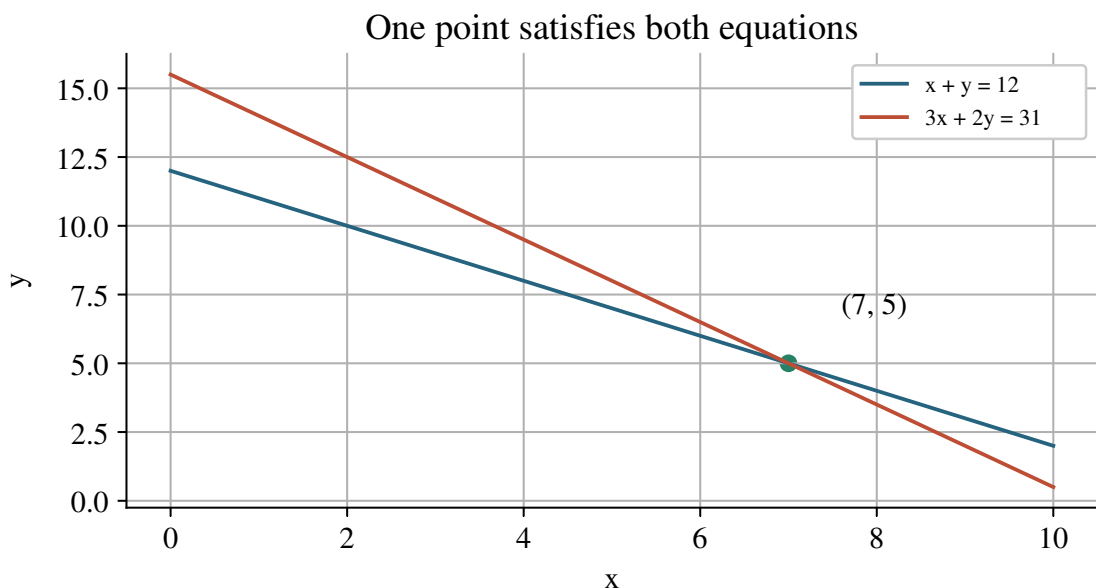
Work through a checked case

$$x + y = 12, \quad 3x + 2y = 31$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If $x+y=12$ and $3x+2y=31$, subtract twice the first equation from the second to obtain $x=7$, then $y=5$. For $y=x+1$ and $y=x^2-1$, $x^2-x-2=0$ gives $x=2$ or -1 , with $y=3$ or 0 .



Test a tempting shortcut

- One equation checked is not enough. A line can meet a quadratic twice, so retain both solutions unless the context removes one. Inequality boundaries may be included or excluded according to the sign.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A pair of simultaneous equations is solved by checking just one of them. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Define the variables and their units. If they count objects, both must be nonnegative integers. For a feasible region, test one point on the required side of each boundary and then take the intersection.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 2. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Combining equations to remove one variable while retaining the same solutions. Choose the relationship, show the method, check its assumptions and interpret the result.

2 · Simultaneous equations and feasible regions · Higher

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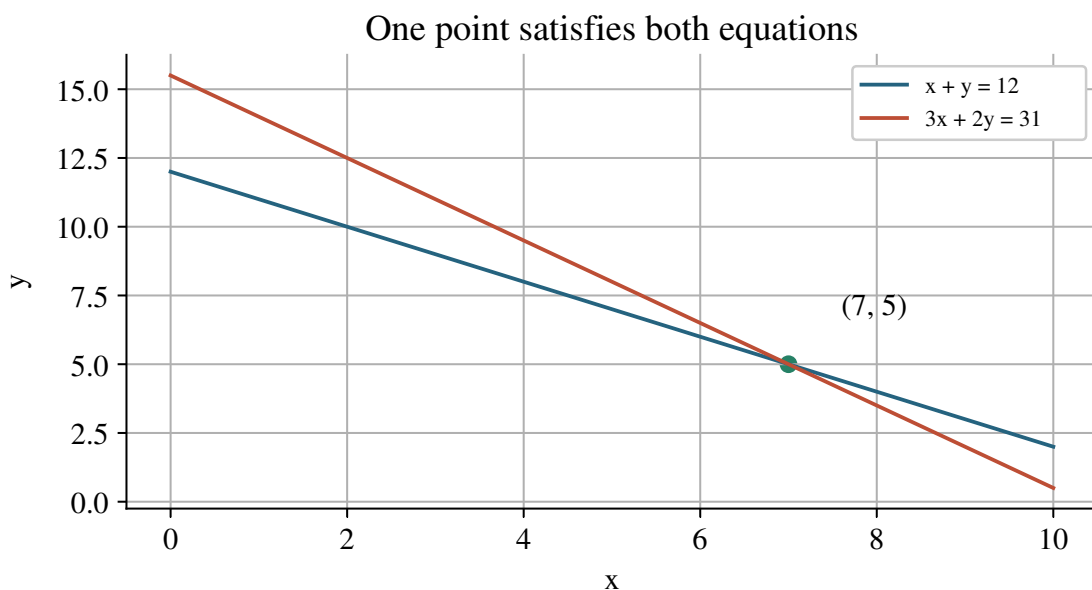
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Interpret a new situation

- Define the variables and their units. If they count objects, both must be nonnegative integers. For a feasible region, test one point on the required side of each boundary and then take the intersection.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 2. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Combining equations to remove one variable while retaining the same solutions. Choose the relationship, show the method, check its assumptions and interpret the result.

3 • Arithmetic sequences and nth terms •

Foundation

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
 - This lesson studies **common difference** 公差: The constant added between consecutive terms of an arithmetic sequence.
-

Choose the mathematical structure

- Find a constant difference for an arithmetic sequence. Its nth term is $a + (n-1)d$. A term-to-term rule describes how to reach the next term; a position-to-term rule gives a term directly.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

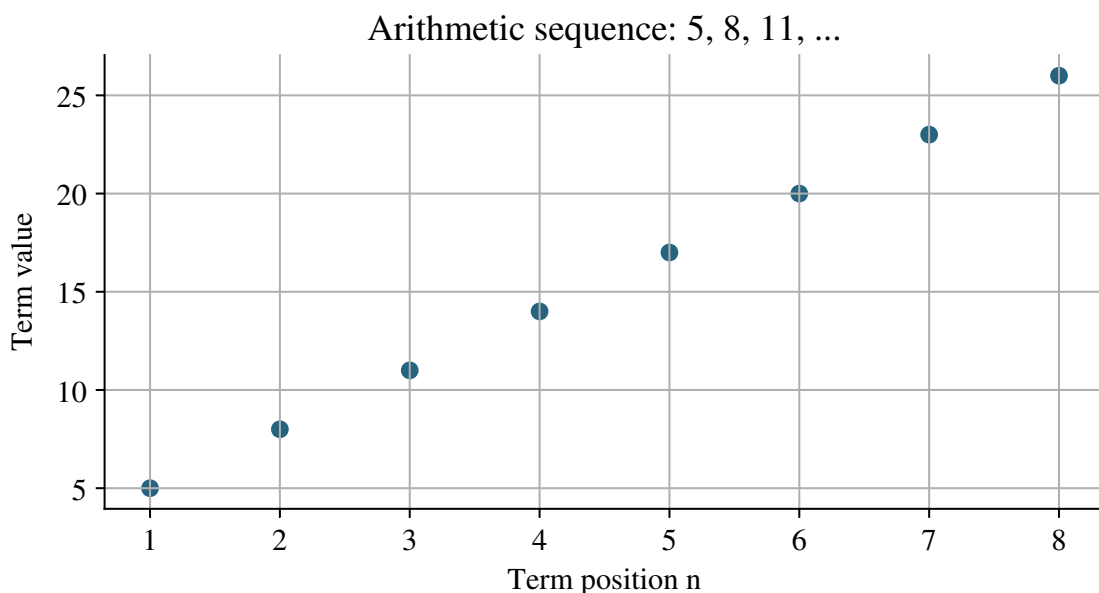
Work through a checked case

$$u_n = a + (n - 1)d$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For 5, 8, 11, 14, ... the common difference is 3. The nth term is $5 + 3(n-1) = 3n + 2$. At $n=8$, $u_8=26$. To find the position of 62, solve $3n+2=62$, giving $n=20$.



Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The first term of a sequence always has index zero. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Generate several terms and check a proposed n th-term rule. Infinite geometric series and advanced sum formulae are excluded from this Foundation/Core lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 3. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The constant added between consecutive terms of an arithmetic sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

3 · Sequences, series and recurrence · Higher

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.

Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

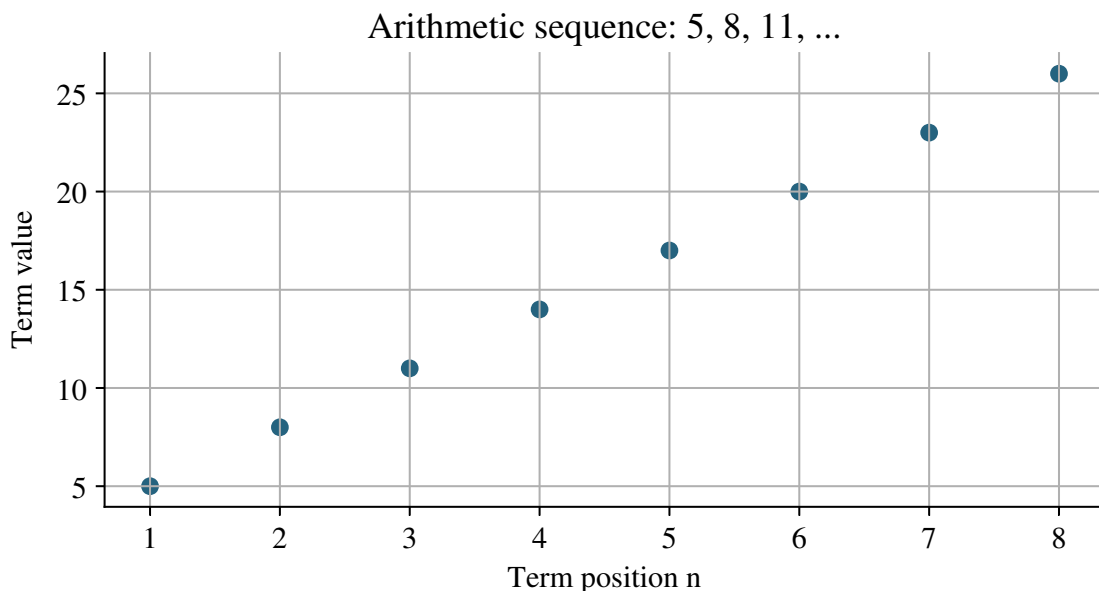
Work through a checked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $a=5, d=3, n=8$, $u_8=5+7\times 3=26$ and $S_8=8(10+21)/2=124$. For $a=12, r=1/2$, $S_{\infty}=12/(1-1/2)=24$. For $u_{(n+1)}=2u_n+1$ with $u_1=1$, the next terms are 3, 7, 15.



Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- 4MA1 · Higher · 3. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

3 · Straight lines and gradients · Foundation

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x.

Choose the mathematical structure

- Gradient is change in y divided by change in x. A straight line has $y=mx+c$, where c is its y-intercept. Parallel lines have equal gradients.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

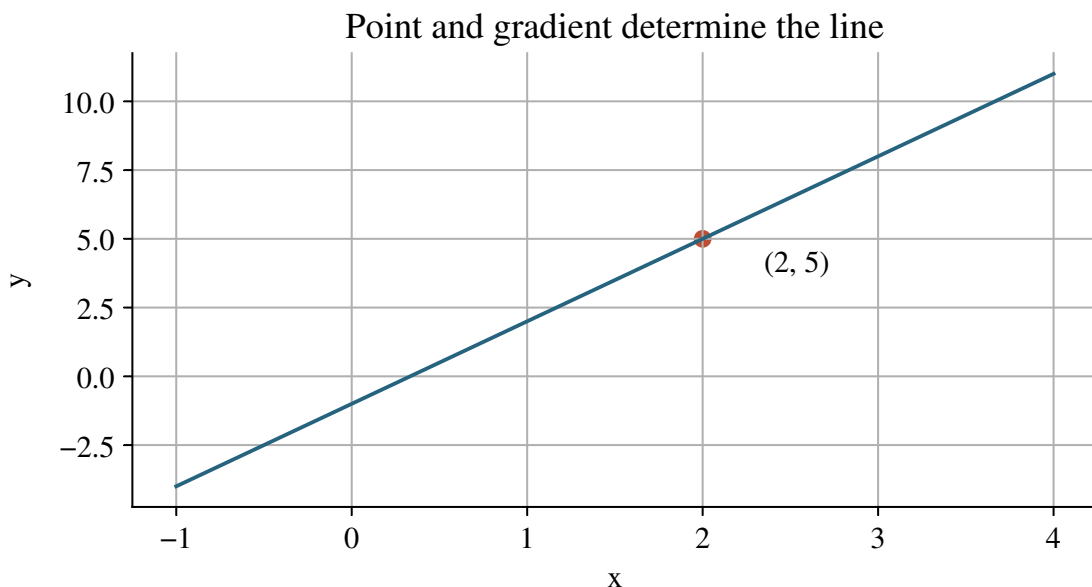
Work through a checked case

$$y = mx + c$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Through (2,5) with gradient 3, substitute to get $5=3\times 2+c$, so $c=-1$ and $y=3x-1$. Points (1,2) and (4,8) give gradient $(8-2)/(4-1)=2$.



Test a tempting shortcut

- A vertical line has no finite gradient; do not force it into $y=mx+c$. Read the signs of a circle's centre carefully. The radius to a tangent is perpendicular to the tangent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Perpendicular nonvertical lines always have equal gradients. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Plot a straight line using two checked points and label its intercept. Perpendicular-gradient formulae and circle equations are not part of this Foundation/Core lesson.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 3. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The change in y divided by the corresponding change in x . Choose the relationship, show the method, check its assumptions and interpret the result.

3 · Coordinate geometry and tangents · Higher

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
 - This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x .
-

Choose the mathematical structure

- A line through (x_1, y_1) with gradient m has $y - y_1 = m(x - x_1)$. Parallel lines have equal gradients. Finite perpendicular gradients multiply to -1 . A circle has $(x - a)^2 + (y - b)^2 = r^2$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

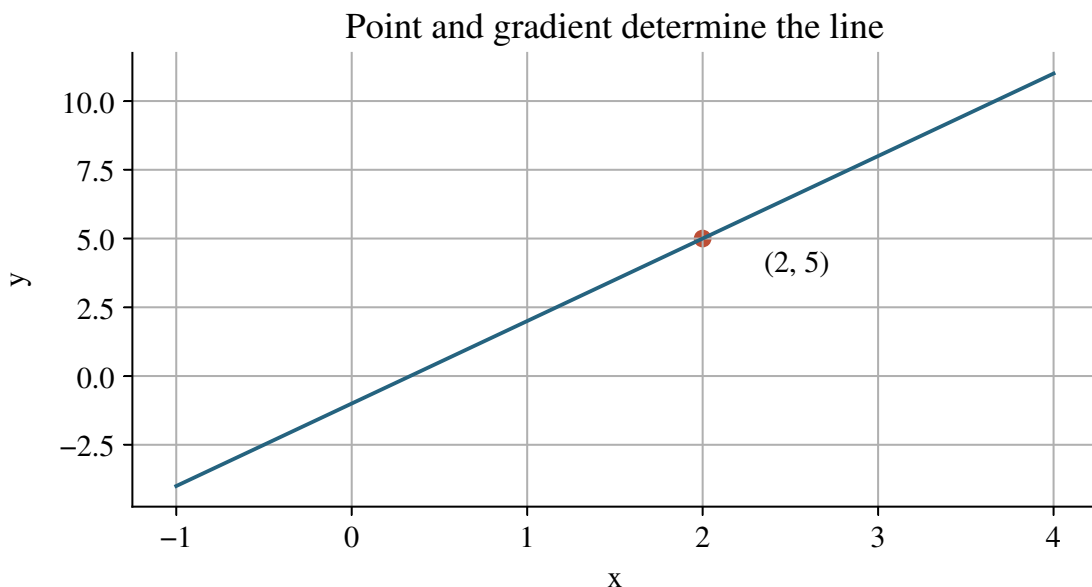
Work through a checked case

$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Test a tempting shortcut

- A vertical line has no finite gradient; do not force it into $y=mx+c$. Read the signs of a circle's centre carefully. The radius to a tangent is perpendicular to the tangent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Perpendicular nonvertical lines always have equal gradients. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections. Substitution produces a quadratic whose discriminant checks the prediction.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 3. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The change in y divided by the corresponding change in x . Choose the relationship, show the method, check its assumptions and interpret the result.

3 · Domains, inverses and composition · Higher

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
 - This lesson studies **domain** 定义域: The set of allowed inputs to a function.
-

Choose the mathematical structure

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

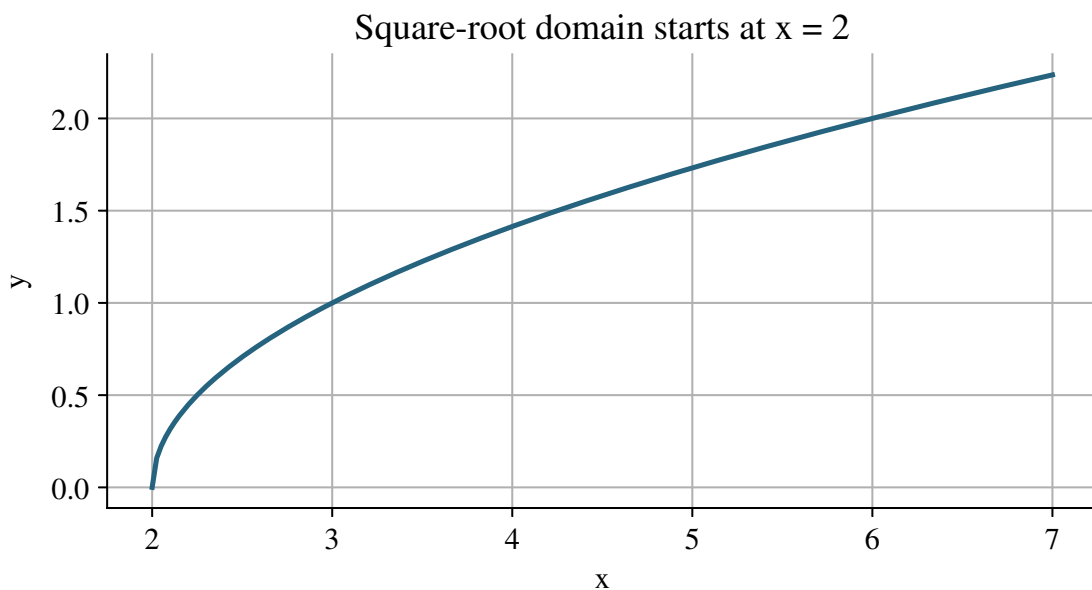
Work through a checked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus $f^{-1}(x)=x^2+2$ with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



Test a tempting shortcut

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 3. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

3 · Derivatives and stationary points · Higher

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
 - This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.
-

Choose the mathematical structure

- For a polynomial term ax^n with nonnegative integer n , the gradient term is $anx^{(n-1)}$. Add the differentiated terms. A stationary point has zero gradient.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

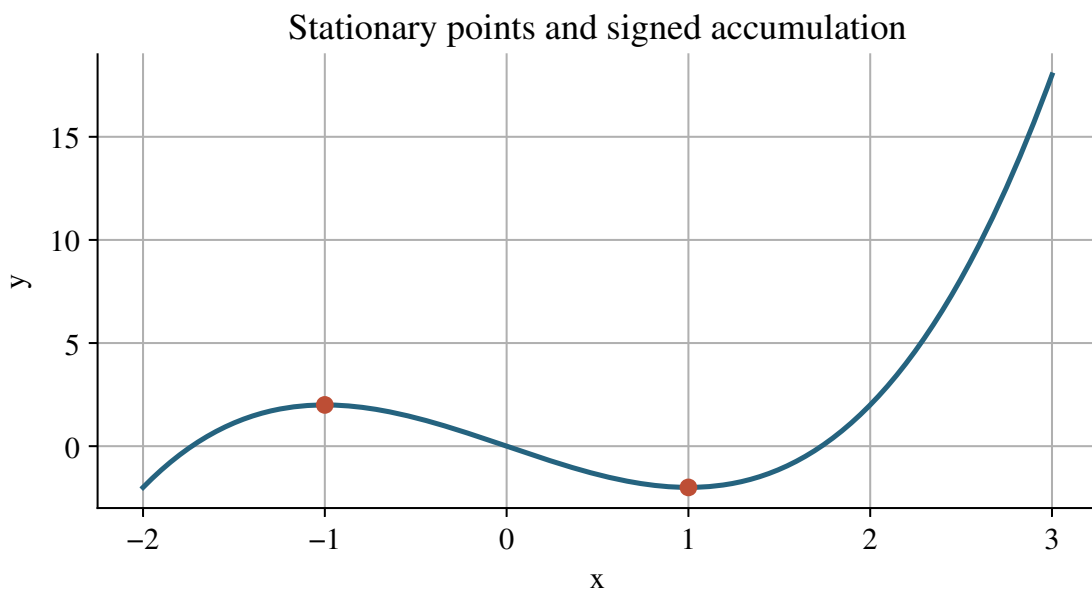
Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.



Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- This advanced-tier IGCSE lesson is limited to polynomial differentiation, tangent gradients and stationary points. Chain, product, quotient and implicit differentiation are excluded.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 3. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

4 · Angles, lengths and area · Foundation

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
 - This lesson studies **scale factor** 相似比: The multiplier that relates corresponding lengths in similar shapes.
-

Choose the mathematical structure

- Use angle facts with a stated reason. Similar shapes have equal corresponding angles and proportional corresponding lengths. Areas of rectangles and triangles come from their dimensions; compound shapes can be split into simpler parts.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$A_{\text{rectangle}} = LW, \quad A_{\text{triangle}} = \frac{1}{2}bh$$

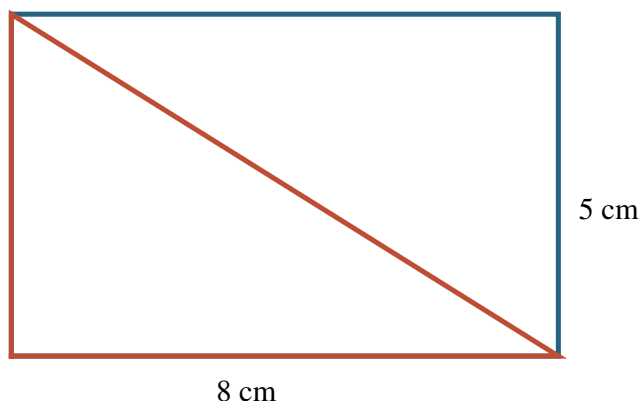
- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A rectangle of length 8 cm and width 5 cm has area $A=LW=40 \text{ cm}^2$. A triangle on the same base and height has area $A=bh/2=20 \text{ cm}^2$. For a pentagon, the interior-angle sum is $(5-2) \times 180 = 540^\circ$.

Same base and height, half the area

Rectangle: 40 cm^2 ; triangle: 20 cm^2



Test a tempting shortcut

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A triangle and rectangle with the same base and height have the same area. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use a labelled sketch and appropriate units. This Foundation/Core lesson does not test area/volume scale factors or advanced circle-theorem proofs.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

4 • Angle reasoning, similarity and mensuration • Higher

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
 - This lesson studies **scale factor** 相似比: The multiplier that relates corresponding lengths in similar shapes.
-

Choose the mathematical structure

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 . State angle reasons explicitly. A circle's tangent is perpendicular to the radius at the contact point.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

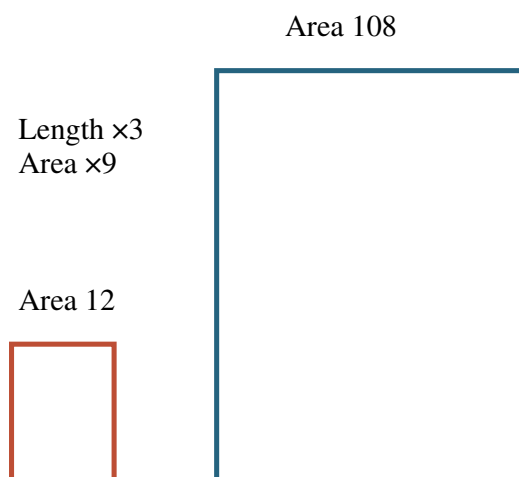
$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



Test a tempting shortcut

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Doubling every length of a solid doubles its volume. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

4 · Right triangles and non-right triangles · Foundation

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

Choose the mathematical structure

- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

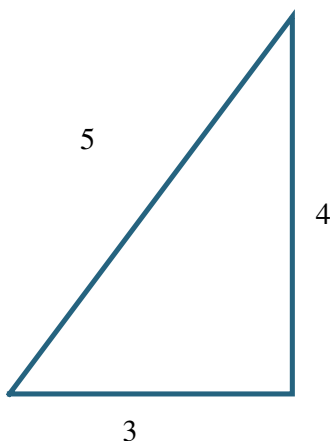
$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. A right triangle with legs 6 and 8 has area $6 \times 8 / 2 = 24$.

Label the known sides before choosing a rule



Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- This Foundation/Core lesson uses right-angled triangles only. Sine and cosine rules for non-right triangles belong to the advanced-tier lesson.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- 4MA1 · Foundation · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

4 · Right triangles and non-right triangles · Higher

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

Choose the mathematical structure

- In a right triangle $a^2 + b^2 = c^2$; $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$. For other triangles, use the sine or cosine rule, or $\text{area} = \frac{1}{2}ab \sin C$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

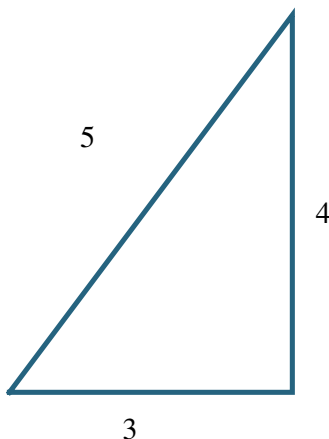
$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

4 · Circle theorems and reasoned proofs · Higher

Which angles see the same chord?

- Two observers see the same chord from the circle. Their angles are linked by a theorem rather than by the apparent size of the drawing.
 - This lesson studies **cyclic quadrilateral** 圆内接四边形: A quadrilateral whose four vertices lie on one circle.
-

Choose the mathematical structure

- The angle at the centre is twice the angle at the circumference on the same arc. Angles in the same segment are equal. Opposite angles of a cyclic quadrilateral sum to 180° . A radius is perpendicular to a tangent at contact.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

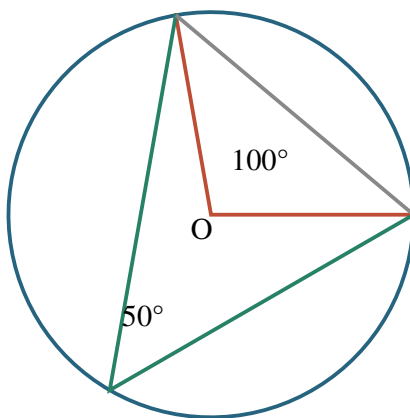
$$\theta_{\text{centre}} = 2\theta_{\text{circumference}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If a central angle is 100° , the corresponding angle at the circumference is 50° . In a cyclic quadrilateral with one angle 112° , its opposite angle is $180-112=68^\circ$. A radius meeting a tangent gives 90° , even if the drawing looks oblique.

The centre angle is twice the circumference angle



Test a tempting shortcut

- Identify the same chord and the correct arc before using a theorem. Two visible right angles do not prove a quadrilateral cyclic without a valid converse argument. A diagram need not be to scale.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every quadrilateral has opposite angles summing to 180° . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Write one reason alongside each angle calculation. For the alternate-segment theorem, name the tangent and chord, then identify the angle in the opposite segment. Use auxiliary radii only when they help the proof.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A quadrilateral whose four vertices lie on one circle. Choose the relationship, show the method, check its assumptions and interpret the result.

4 · Constructions, loci and geometric conditions · Foundation

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
 - This lesson studies **locus** 轨迹: The set of all points satisfying a stated geometric condition.
-

Choose the mathematical structure

- Points equally distant from A and B lie on the perpendicular bisector of AB. Points at fixed distance r from C lie on a circle. Points equally distant from two intersecting lines lie on their angle bisectors.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

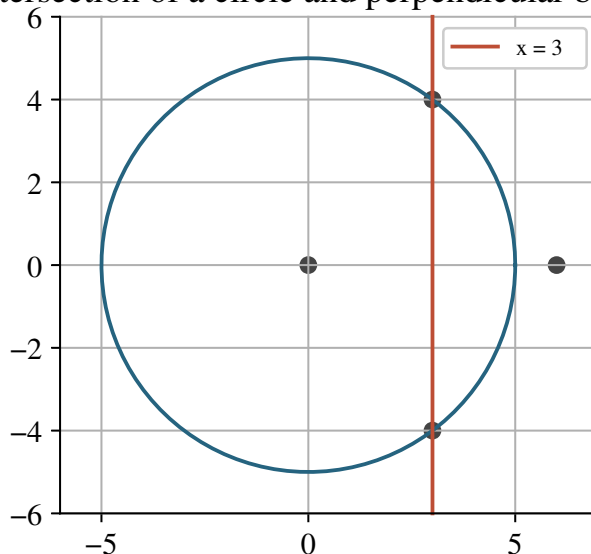
$$x = 3, \quad x^2 + y^2 = 25$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $A=(0,0)$ and $B=(6,0)$, the perpendicular bisector is $x=3$. Points also 5 units from A satisfy $x^2+y^2=25$. Substituting $x=3$ gives $y^2=16$, so $(3,4)$ and $(3,-4)$ satisfy both conditions.

Intersection of a circle and perpendicular bisector



Test a tempting shortcut

- The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines. A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Points equally distant from two points always lie on their angle bisector. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Translate each condition into a locus before finding intersections. For a region closer to A than B, choose the correct side of the perpendicular bisector and show whether a boundary is allowed.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The set of all points satisfying a stated geometric condition. Choose the relationship, show the method, check its assumptions and interpret the result.

4 · Constructions, loci and geometric conditions · Higher

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
 - This lesson studies **locus** 轨迹: The set of all points satisfying a stated geometric condition.
-

Choose the mathematical structure

- Points equally distant from A and B lie on the perpendicular bisector of AB. Points at fixed distance r from C lie on a circle. Points equally distant from two intersecting lines lie on their angle bisectors.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

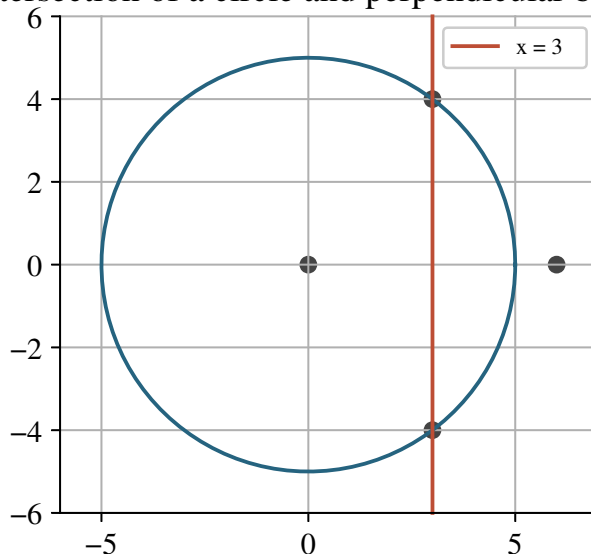
$$x = 3, \quad x^2 + y^2 = 25$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $A=(0,0)$ and $B=(6,0)$, the perpendicular bisector is $x=3$. Points also 5 units from A satisfy $x^2+y^2=25$. Substituting $x=3$ gives $y^2=16$, so $(3,4)$ and $(3,-4)$ satisfy both conditions.

Intersection of a circle and perpendicular bisector



Test a tempting shortcut

- The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines. A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Points equally distant from two points always lie on their angle bisector. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Translate each condition into a locus before finding intersections. For a region closer to A than B, choose the correct side of the perpendicular bisector and show whether a boundary is allowed.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 4. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The set of all points satisfying a stated geometric condition. Choose the relationship, show the method, check its assumptions and interpret the result.

5 · Vectors and transformation geometry · Foundation

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.
-

Choose the mathematical structure

- Add corresponding vector components and subtract position vectors to find a displacement. A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

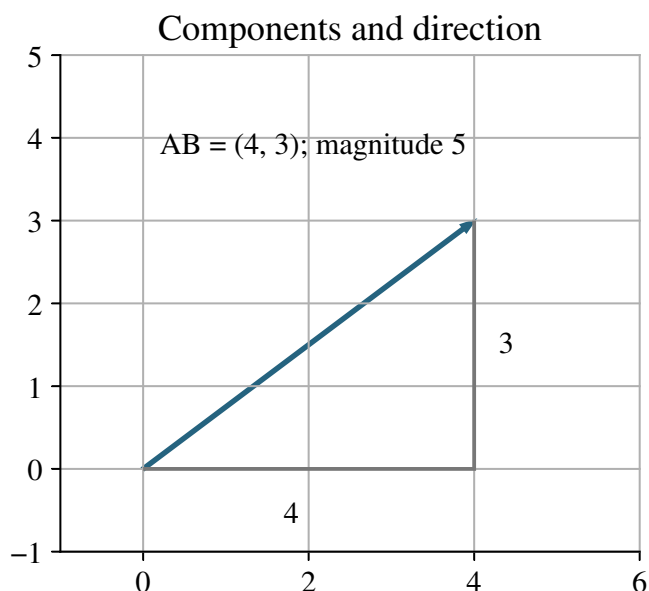
Work through a checked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. From $A=(1,2)$ to $B=(5,5)$, displacement $\overrightarrow{AB}=(4,3)$. Its magnitude is $\sqrt{4^2+3^2}=5$.



Test a tempting shortcut

- The order of subtraction matters: $BA = -AB$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

AB and BA always have the same components. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Draw arrows with direction and identify the starting and ending points. Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 5. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

5 · Vectors and transformation geometry · Higher

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.

Choose the mathematical structure

- Add corresponding vector components and subtract position vectors to find a displacement. A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

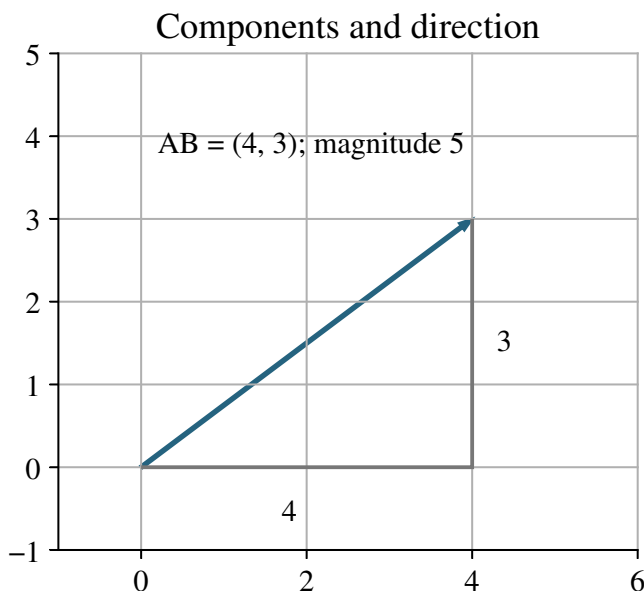
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$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

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Worked example

With $a=(4,1)$ and $b=(1,3)$, $a+b=(5,4)$. From $A=(1,2)$ to $B=(5,5)$, displacement $AB=(4,3)$. Its magnitude is $\sqrt{4^2+3^2}=5$.



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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

5 · Reflections, rotations and enlargements · Foundation

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
 - This lesson studies **centre of enlargement** 位似中心: The point from which each point's displacement is multiplied by a scale factor.
-

Choose the mathematical structure

- A translation adds a vector. A reflection reverses signed perpendicular distance from a mirror line. A rotation needs a centre, angle and direction. For enlargement from C, use new $P = C + k(P - C)$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

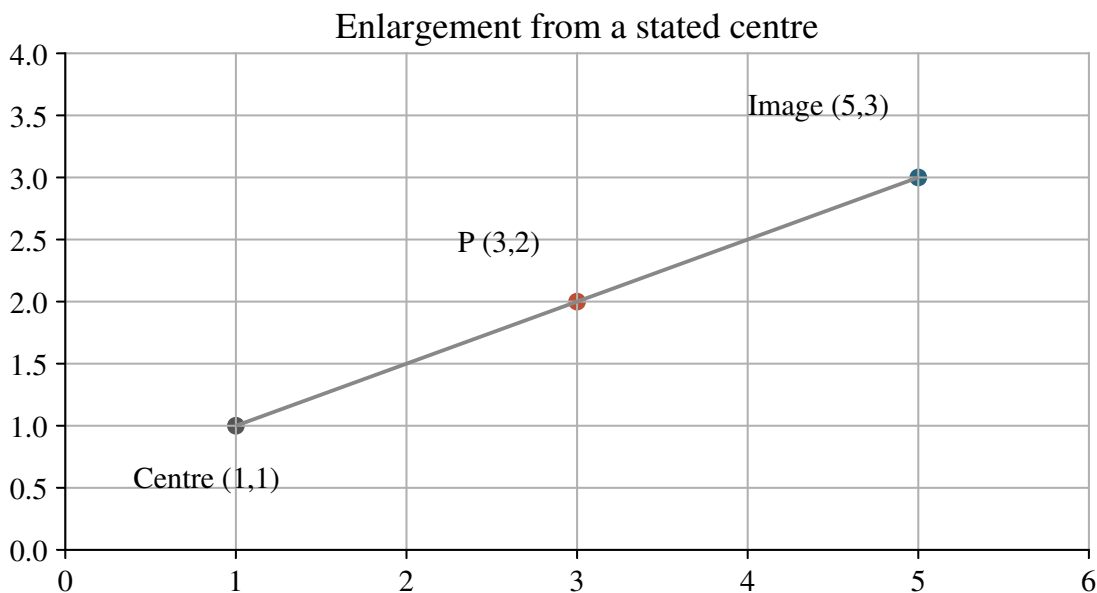
Work through a checked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $C=(1,1), P=(3,2), k=2$, $P-C=(2,1)$, so new $P=(1,1)+2(2,1)=(5,3)$. Reflection of $(3,2)$ in the y-axis gives $(-3,2)$. A 90° anticlockwise rotation about the origin gives $(-2,3)$.



Test a tempting shortcut

- A rotation needs its centre and direction, not just an angle. A negative enlargement factor places the image on the opposite side of the centre. A translation does not change orientation or size.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every enlargement is centred at the origin unless its scale factor is negative. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Describe a transformation completely before constructing the image. Check corresponding distances and angles. For combined transformations, apply them in the stated order; they usually do not commute.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 5. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The point from which each point's displacement is multiplied by a scale factor. Choose the relationship, show the method, check its assumptions and interpret the result.

5 · Reflections, rotations and enlargements · Higher

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
 - This lesson studies **centre of enlargement** 位似中心: The point from which each point's displacement is multiplied by a scale factor.
-

Choose the mathematical structure

- A translation adds a vector. A reflection reverses signed perpendicular distance from a mirror line. A rotation needs a centre, angle and direction. For enlargement from C, use new $P = C + k(P - C)$.

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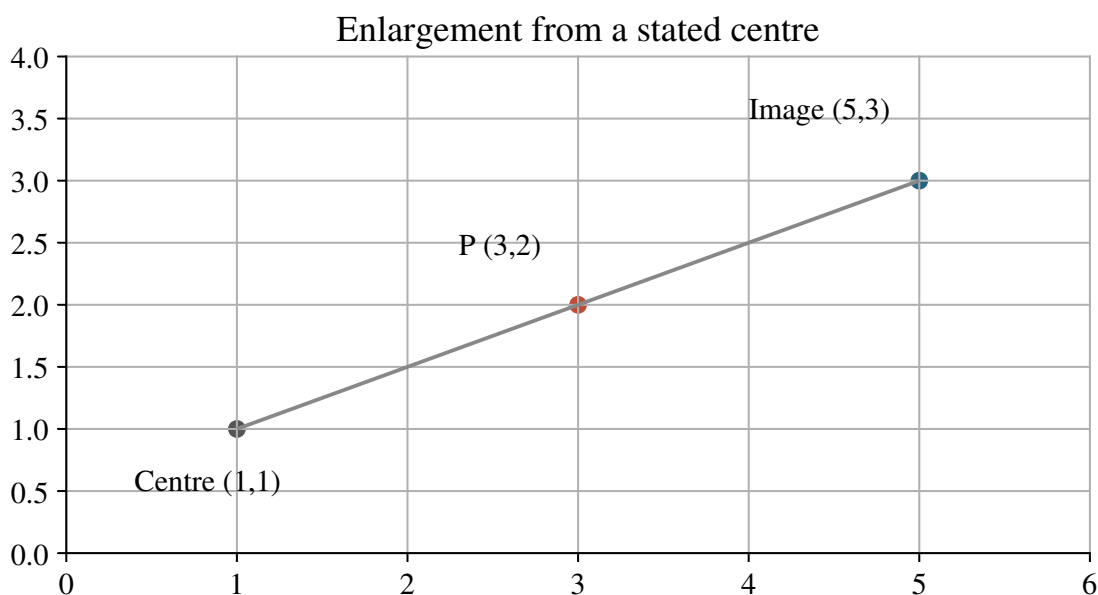
Work through a checked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

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Use this in your course

- 4MA1 · Higher · 5. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The point from which each point's displacement is multiplied by a scale factor. Choose the relationship, show the method, check its assumptions and interpret the result.

6 · Centre, spread and data displays · Foundation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
 - This lesson studies **range** 极差: The maximum observed value minus the minimum observed value.
-

Choose the mathematical structure

- The mean is total divided by count. The median is the central value after sorting. The range is maximum minus minimum. Use frequency tables, bar charts and suitable comparisons.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

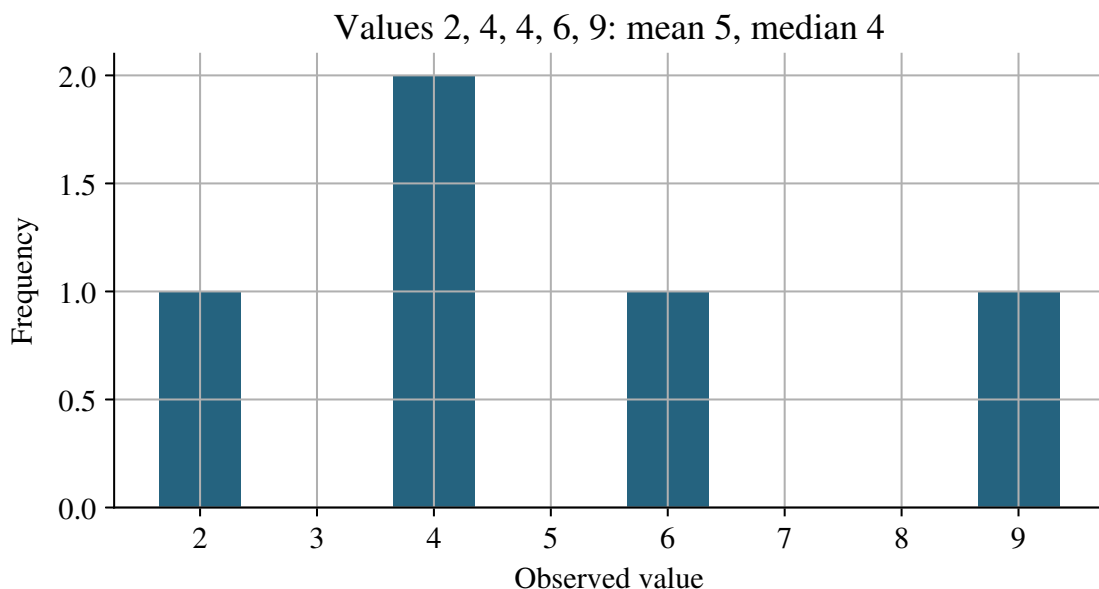
Work through a checked case

$$\bar{x} = \frac{\sum x_i}{n}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For 2,4,4,6,9, total=25 and count=5, so mean=5. The central value is 4, so median=4. The range is 9-2=7. Explain both a typical value and the spread.



Test a tempting shortcut

- The tallest histogram bar need not contain the most observations. A grouped mean is an estimate. Correlation does not prove causation, and extrapolation extends beyond the observed range.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The mean of a list must always be one of its observed values. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- A bar chart uses separate bars for categories. Unequal-class-width histograms and formal density calculations are outside this Foundation/Core lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 6. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The maximum observed value minus the minimum observed value. Choose the relationship, show the method, check its assumptions and interpret the result.

6 · Data summaries, histograms and interpretation

· Higher

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
 - This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.
-

Choose the mathematical structure

- Compare an appropriate average and spread in context. A histogram uses area for frequency, so height=frequency/class width. Grouped estimates assume representative values within intervals.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

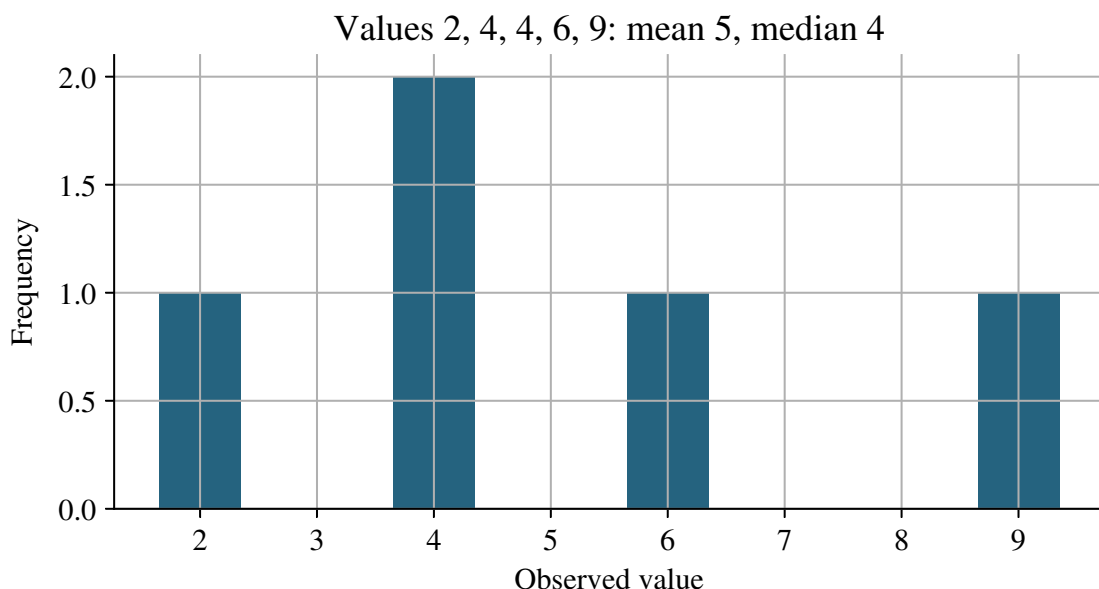
Work through a checked case

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A class from 10 to 20 with frequency 30 has density $30/10=3$. A class from 20 to 40 with frequency 20 has density $20/20=1$. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



Test a tempting shortcut

- The tallest histogram bar need not contain the most observations. A grouped mean is an estimate. Correlation does not prove causation, and extrapolation extends beyond the observed range.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A histogram bar's height always equals its frequency, even with unequal class widths. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a display that fits the data type. Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 6. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Frequency divided by class width, used as histogram height. Choose the relationship, show the method, check its assumptions and interpret the result.

6 · Probability trees and outcomes · Foundation

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

Choose the mathematical structure

- A probability lies between 0 and 1. Exhaustive, mutually exclusive outcomes have probabilities summing to 1. Multiply successive branch probabilities and add separate routes to an outcome.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

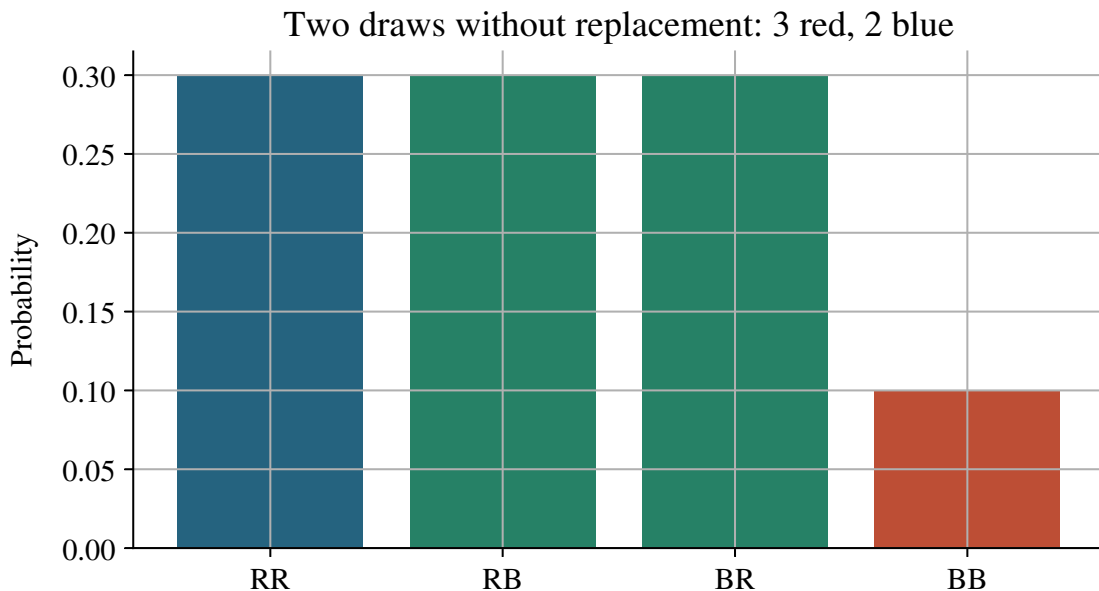
Work through a checked case

$$P(RR) = P(R_1)P(R_2 | R_1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

A bag contains 3 red and 2 blue counters. With replacement, $P(\text{two red}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} = 0.36$. Without replacement, the red-red branch is $\frac{3}{5} \times \frac{2}{4} = 0.3$. Label each branch before multiplying.



Test a tempting shortcut

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use a frequency table or a simple tree before calculating. Formal conditional probability formulae are outside this Foundation/Core support lesson.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Foundation · 6. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

6 · Probability, trees and conditional reasoning · Higher

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
 - This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.
-

Choose the mathematical structure

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

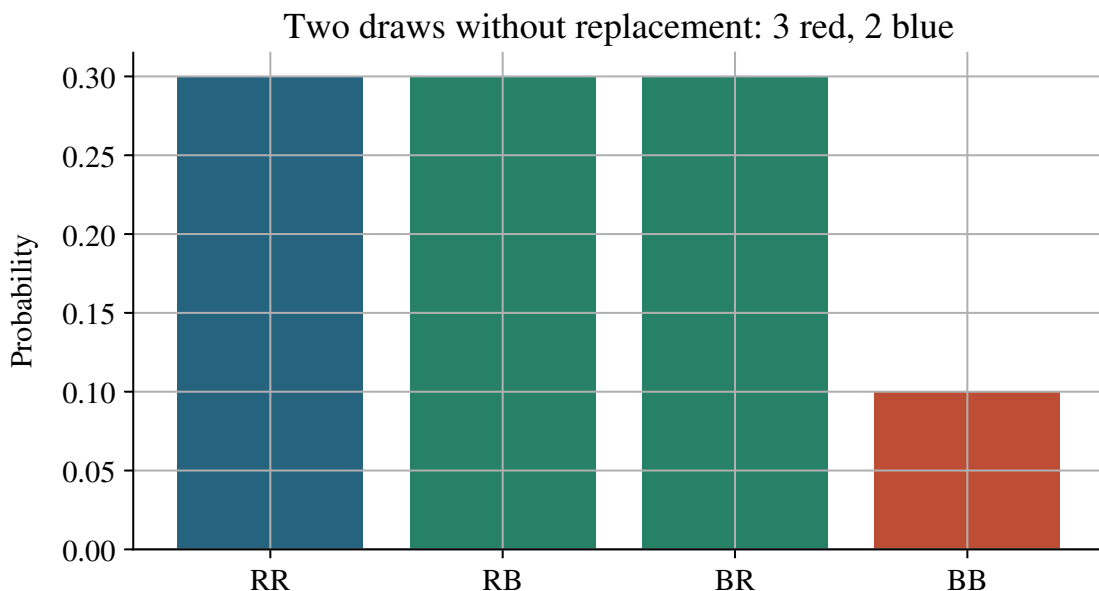
Work through a checked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



Test a tempting shortcut

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 6. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

6 · Cumulative frequency and box plots · Higher

Which route is more consistent?

- Two bus routes have similar typical times but different reliability. Quartiles show the middle half of the journeys.
 - This lesson studies **interquartile range** 四分位距: The difference between the upper and lower quartiles.
-

Choose the mathematical structure

- A cumulative frequency counts observations below successive class boundaries. Read quartiles at one quarter, one half and three quarters of the total frequency. A box plot represents minimum, lower quartile, median, upper quartile and maximum.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

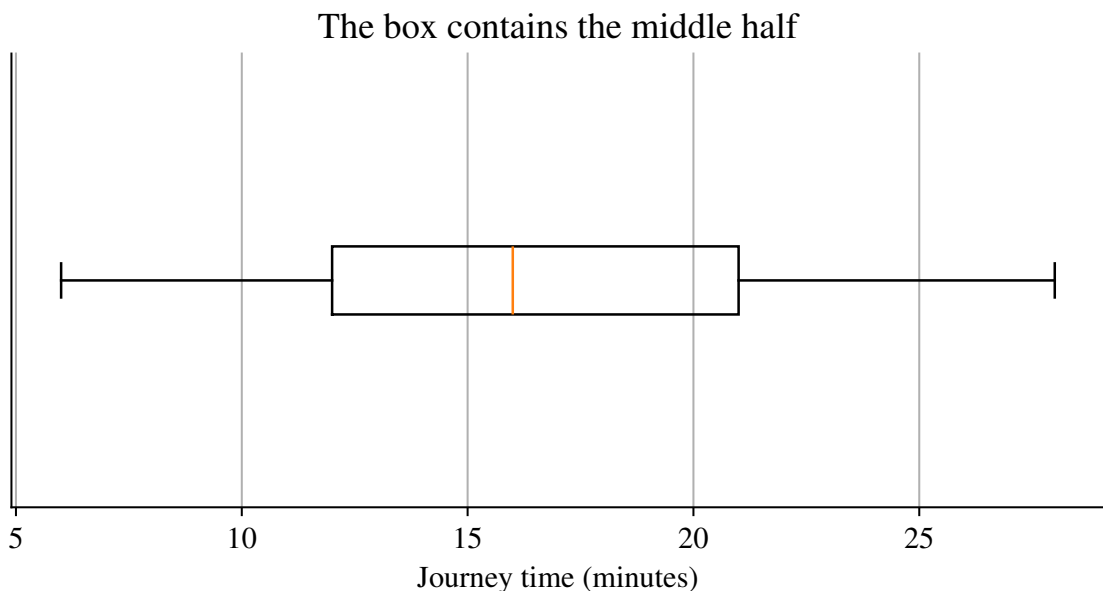
Work through a checked case

$$\text{IQR} = Q_3 - Q_1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For 80 observations, read Q_1 at cumulative frequency 20, median at 40 and Q_3 at 60. If $Q_1=12$, $Q_3=21$, then $\text{IQR}=9$. Compare the medians for typical journey time and the IQRs for consistency.



Test a tempting shortcut

- Plot against class boundaries rather than midpoints. Grouped quartiles are estimates. The range is sensitive to extremes; the IQR describes only the middle half.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The interquartile range always equals the maximum minus the minimum. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain a comparison in the context of the measured quantity. An outlier rule may use $Q1 - 1.5IQR$ and $Q3 + 1.5IQR$; use the rule specified in the task rather than assuming every graph follows it.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- 4MA1 · Higher · 6. Match the target tier and specification before assigning extensions.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The difference between the upper and lower quartiles. Choose the relationship, show the method, check its assumptions and interpret the result.