

Notes

Pearson Edexcel · International A-Level · Pure Mathematics

YPM01: course teaching notes

Version: Issue 3, April 2019; first teaching 2018; unit assessment from 2019

These are original course-owned teaching notes. Objective-level exceptions are listed in the review, and lessons are tier labelled.

P1 · Indices, surds and standard form

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
 - This lesson studies **index** 指数: The power to which a base is raised.
-

Choose the mathematical structure

- For the same positive base, multiplication adds indices and division subtracts them. A negative index means reciprocal; a fractional index represents a root. Standard form has $1 \leq a < 10$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

Test a tempting shortcut

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 . Do not round a surd when an exact answer is requested.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For every positive a , $a^2 + a^3$ equals a^5 . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check powers of ten against the original quantity. Use surds for exact geometry, and round only the final length when the question asks for a decimal.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL pure mathematics; official unit P1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The power to which a base is raised. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
 - This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
-

Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
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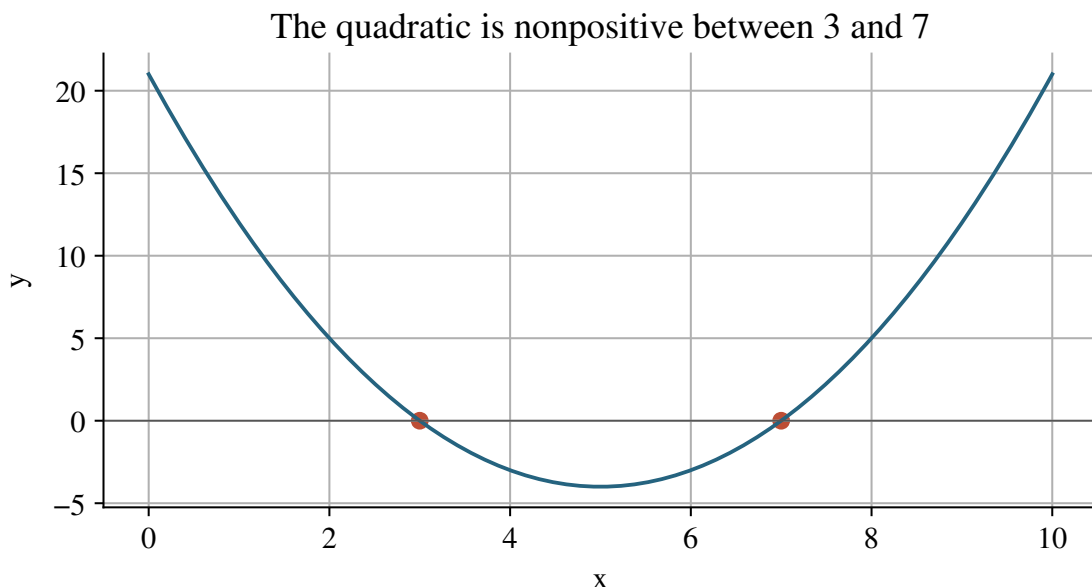
Work through a checked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Key idea

The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
 - This lesson studies **gradient** 斜率: The change in y divided by the corresponding change in x .
-

Choose the mathematical structure

- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$. Parallel lines have equal gradients. Finite perpendicular gradients multiply to -1 . A circle has $(x-a)^2+(y-b)^2=r^2$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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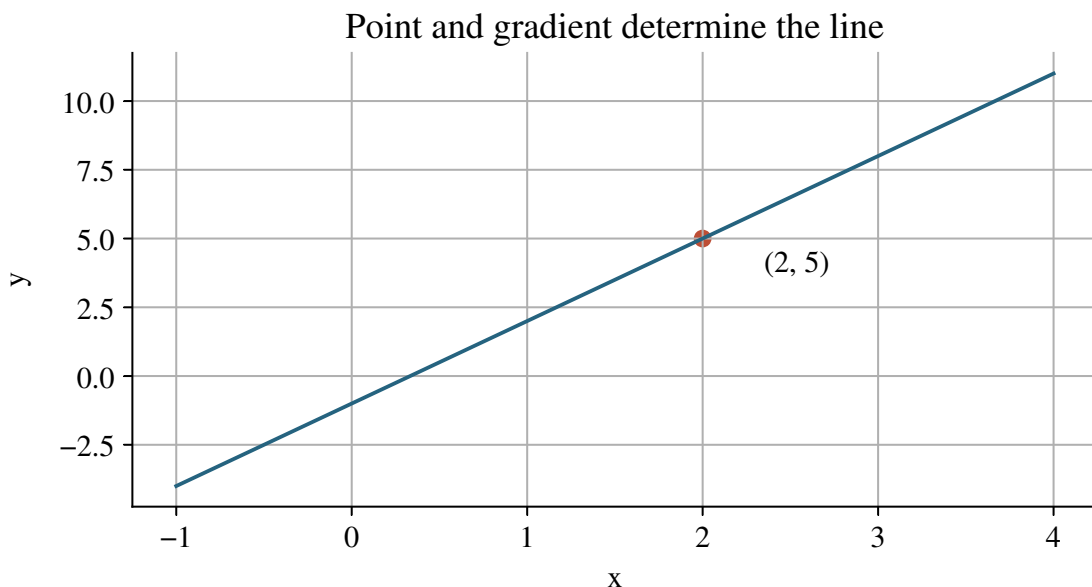
Work through a checked case

$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Test a tempting shortcut

- A vertical line has no finite gradient; do not force it into $y=mx+c$. Read the signs of a circle's centre carefully. The radius to a tangent is perpendicular to the tangent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Perpendicular nonvertical lines always have equal gradients. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections. Substitution produces a quadratic whose discriminant checks the prediction.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The change in y divided by the corresponding change in x . Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Right triangles and non-right triangles

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
 - This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.
-

Choose the mathematical structure

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$. For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

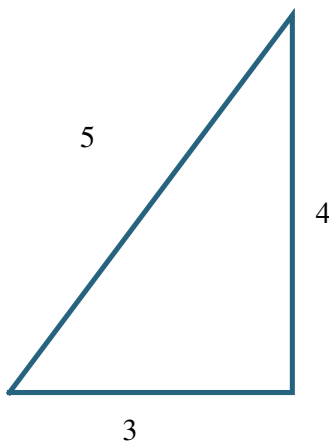
$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.
-

Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

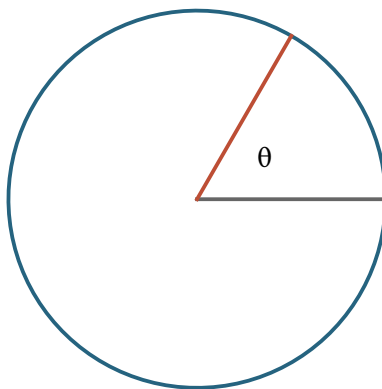
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x = 0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.

- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

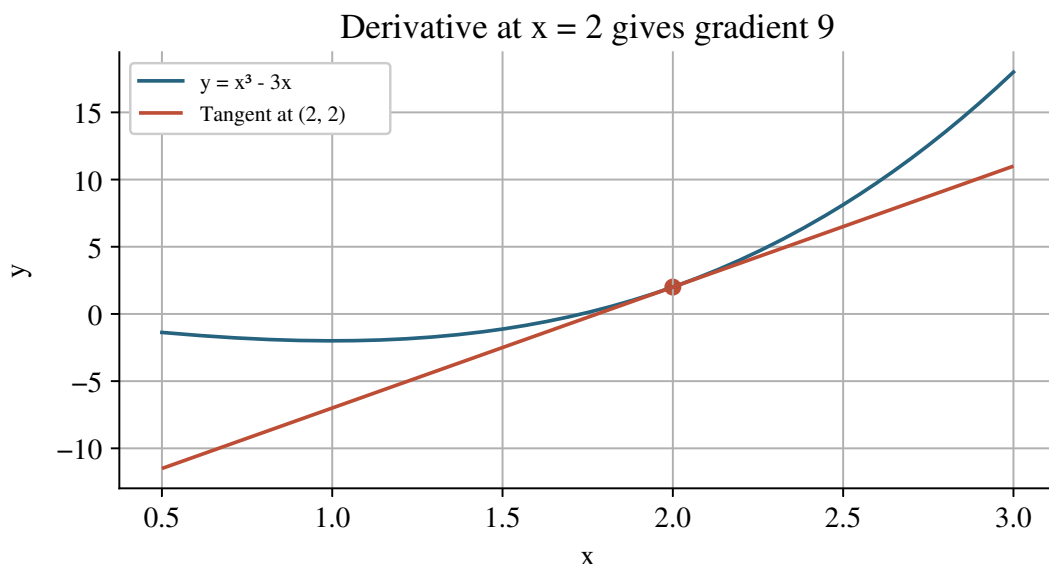
Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad m = f'(x_0)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y = x^3 - 3x$, $dy/dx = 3x^2 - 3$. At $x = 2$, the gradient is 9 and $y = 2$. The tangent is $y - 2 = 9(x - 2)$. The normal gradient is $-1/9$.



Test a tempting shortcut

- A derivative gives a gradient, not the ordinate. Evaluate y and dy/dx separately at the supplied x -coordinate; the normal gradient is the negative reciprocal of a nonzero tangent gradient.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A tangent and normal at the same point always have the same gradient. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P1 covers polynomial derivatives, gradients, tangents and normals. Optimization and stationary-point classification are taught in P2; they are not introduced here.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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Key idea

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

P1 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
 - This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.
-

Choose the mathematical structure

- Integrate polynomial powers by increasing the index by one and dividing by the new index. Include an arbitrary constant and use a supplied point to determine it.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

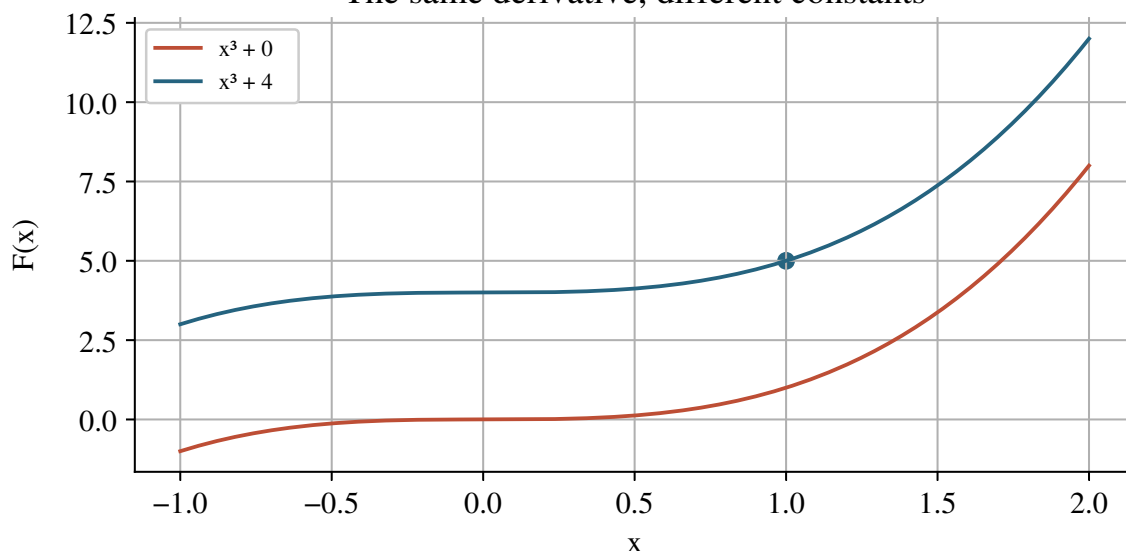
$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

An antiderivative of $3x^2$ is $x^3 + C$ because differentiating x^3 gives $3x^2$. If $F(1)=5$, then $1+C=5$, so $C=4$ and $F(x)=x^3+4$.

The same derivative, different constants



Test a tempting shortcut

- An indefinite integral needs a constant. Integrating each term changes its power; copying the derivative rule gives the wrong result.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every indefinite integral has only one possible antiderivative. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P1 uses indefinite polynomial integration and a point to determine the constant. Definite integrals, areas and the trapezium rule are P2 content.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

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Key idea

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.
-

Choose the mathematical structure

- A deductive proof starts from stated definitions or assumptions. A counterexample refutes an all-values claim. Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

Odd + odd: $(2a + 1) + (2b + 1)$

Factor 2: $2(a + b + 1)$

The bracket is an integer $\rightarrow$ the sum is even

Test a tempting shortcut

- Do not assume the conclusion while proving it. In contradiction, identify the impossible consequence and reject the initial contrary assumption. Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Checking the first ten integers proves a claim for every positive integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- AQA proof includes deduction, exhaustion and contradiction. IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction. Keep the method matched to the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Key idea

A single valid case that disproves a universal claim. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
 - This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
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Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
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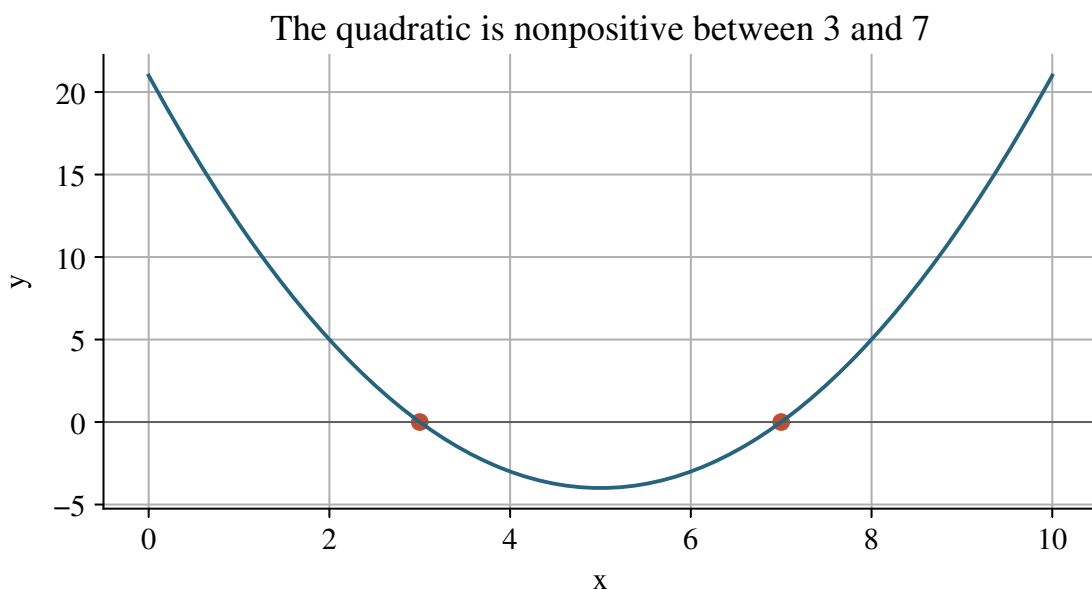
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$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2 - 10x + 21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
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A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.
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The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Coordinate geometry and tangents

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
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Choose the mathematical structure

- A line through (x_1, y_1) with gradient m has $y-y_1=m(x-x_1)$. Parallel lines have equal gradients. Finite perpendicular gradients multiply to -1 . A circle has $(x-a)^2+(y-b)^2=r^2$.
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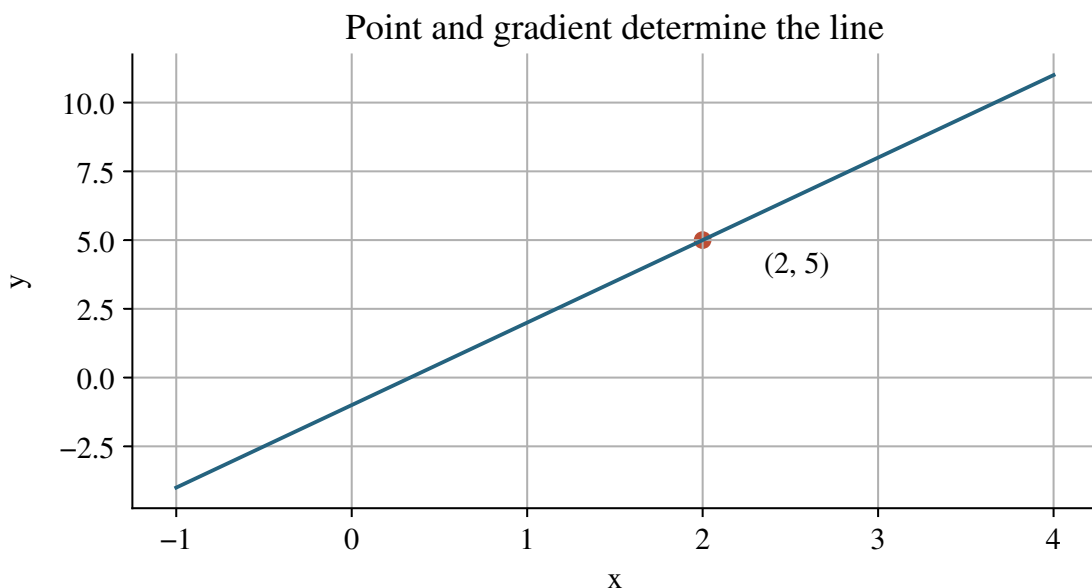
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$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

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Worked example

Through (2,5) with gradient 3, $y-5=3(x-2)$, so $y=3x-1$. A perpendicular through the same point has $y-5=-(x-2)/3$. The circle $(x-2)^2+(y+1)^2=25$ has centre (2,-1) and radius 5.



Test a tempting shortcut

- A vertical line has no finite gradient; do not force it into $y=mx+c$. Read the signs of a circle's centre carefully. The radius to a tangent is perpendicular to the tangent.
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Perpendicular nonvertical lines always have equal gradients. This claim is false. Explain which definition or assumption it violates.

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The change in y divided by the corresponding change in x . Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Sequences, series and recurrence

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
 - This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.
-

Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

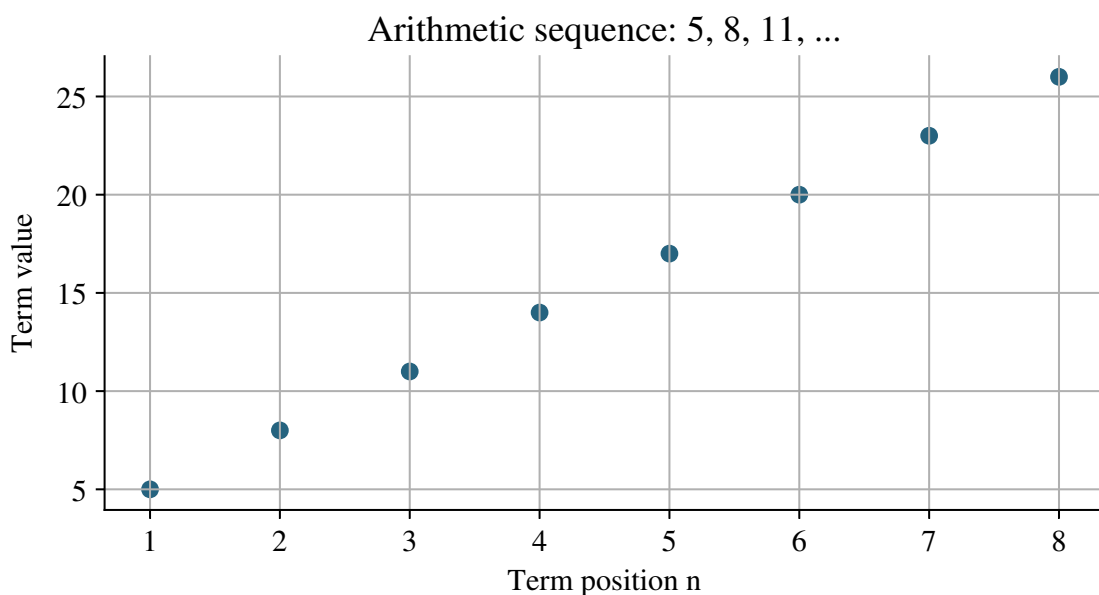
Work through a checked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10+21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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Key idea

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

Choose the mathematical structure

- For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+\frac{n(n-1)}{2}x^2+\dots$ with $|x|<1$. Factor out constants before using this form.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5$. For $(1+x)^{-1}$, the first three terms are $1-x+x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1\approx 0.909091$.

$$(1+2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

Test a tempting shortcut

- An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$. Positive-integer expansions are finite and do not have that convergence restriction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.

- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

Choose the mathematical structure

- For $y=Ae^{kt}$, k is a proportional rate. Taking logs gives $\ln y = \ln A + kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

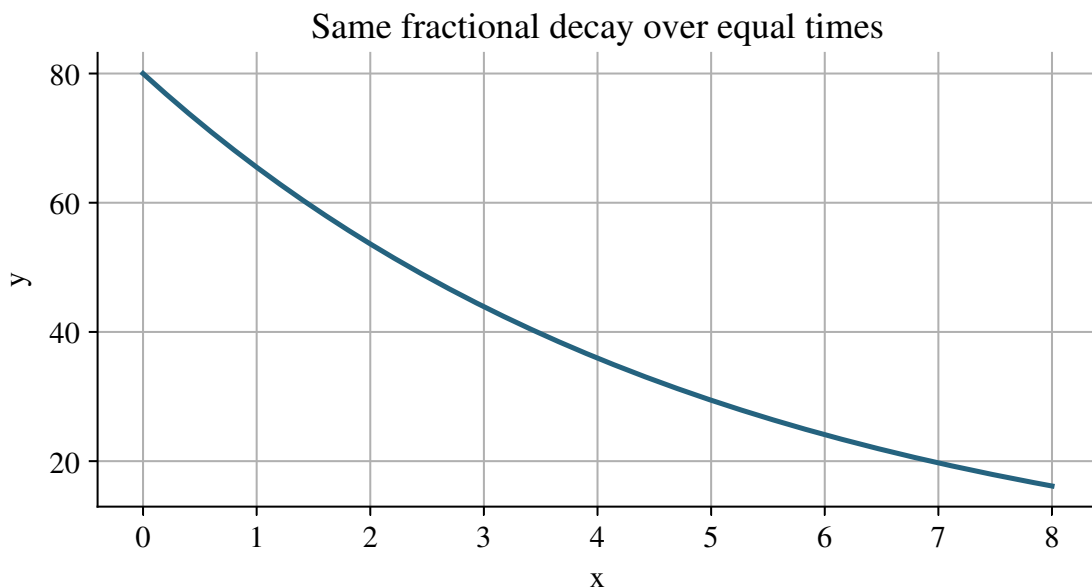
Work through a checked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If $y=80e^{(-0.2t)}$, $y=40$ gives $e^{(-0.2t)}=1/2$. Hence $t=\ln 2/0.2 \approx 3.466$. For $3^x=20$, $x=\ln 20/\ln 3$. Plotting $\ln y$ against t linearises this exponential model.



Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For positive x and y , $\ln(x+y) = \ln x + \ln y$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.
-

Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

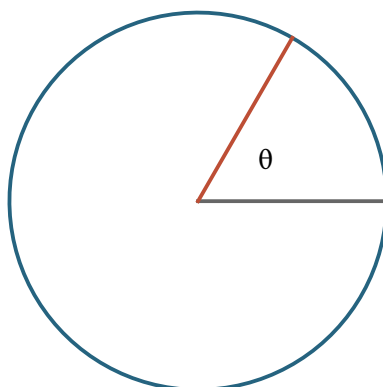
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

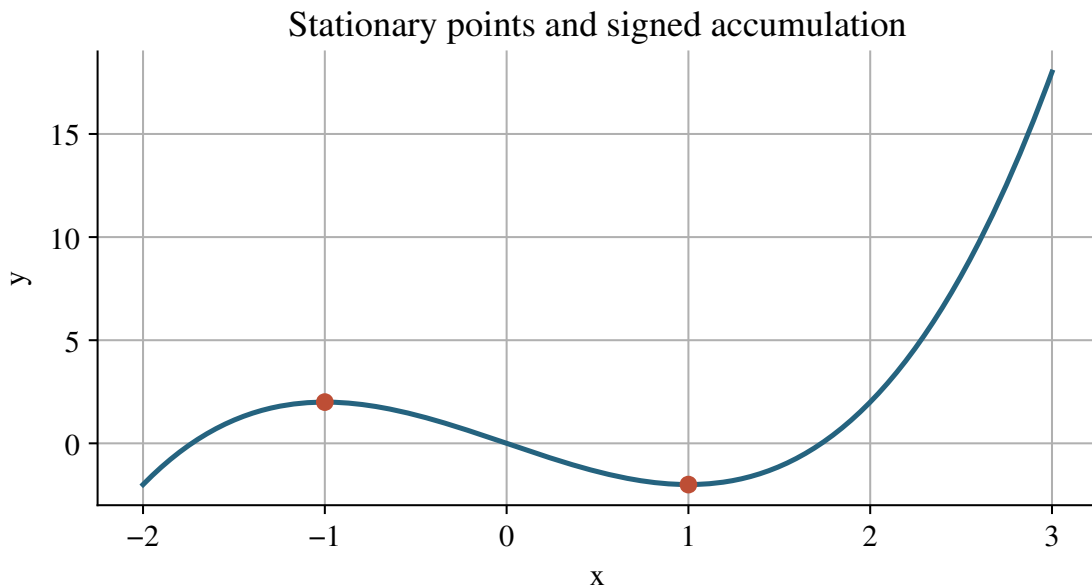
Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.



Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
 - This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.
-

Choose the mathematical structure

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$. A definite integral is signed accumulation. Split at sign changes when total area or distance is required.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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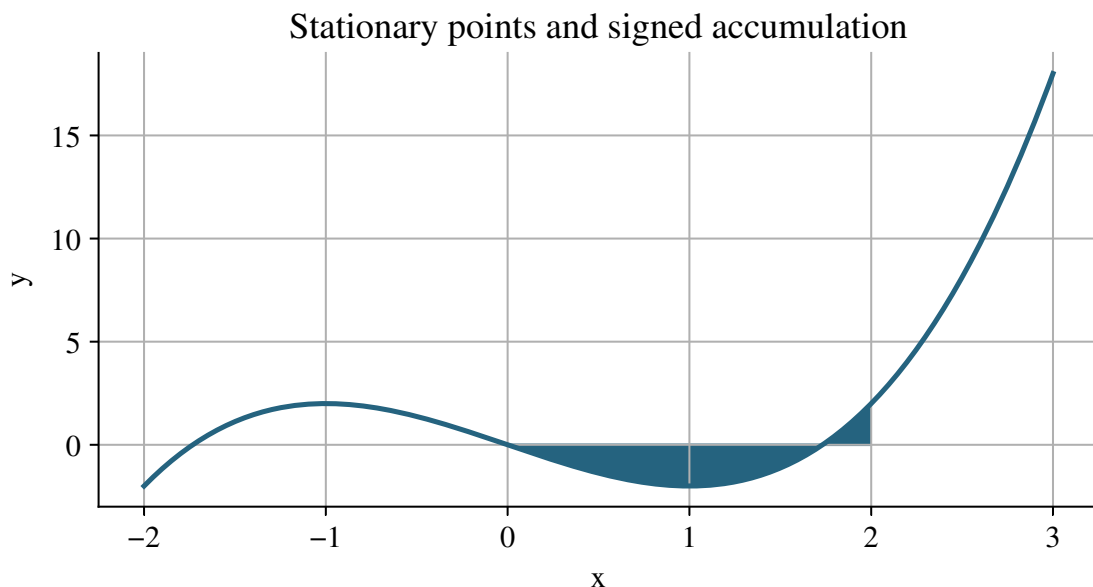
Work through a checked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $v(t) = 3t^2 - 3$ over $0 \leq t \leq 2$, displacement $= [t^3 - 3t]_0^2 = 2$. Since v changes sign at $t = 1$, distance $= -[t^3 - 3t]_0^1 + [t^3 - 3t]_1^2 = 2 + 4 = 6$. The constants cancel only for a definite integral.



Test a tempting shortcut

- The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$. Signed area can be zero even when the total enclosed area is positive.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

P2 · Root finding and numerical integration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.
-

Choose the mathematical structure

- For equal strip width h , the trapezium estimate is $h/2$ times the sum of the two endpoint heights plus twice the internal heights.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

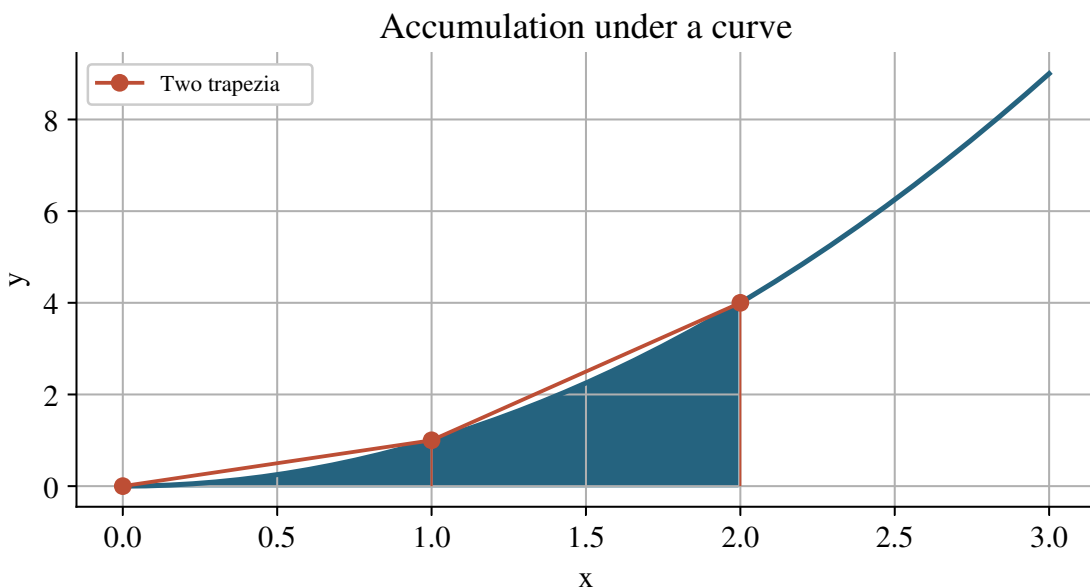
Work through a checked case

$$A \approx \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=x^2$ on $[0,2]$ with two equal strips, $h=1$ and the heights are 0,1,4. The estimate is $(1/2)(0+2\times 1+4)=3$. The exact area is $8/3$, so this convex curve gives an overestimate.



Test a tempting shortcut

- Use equal strip widths and include internal heights twice. Curvature decides whether the estimate is above or below the exact area; being increasing alone does not.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

An increasing function always gives a trapezium overestimate. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P2 introduces the trapezium rule. Newton–Raphson and other root-finding methods belong to other named units; they are not part of this P2 lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
 - This lesson studies **domain** 定义域: The set of allowed inputs to a function.
-

Choose the mathematical structure

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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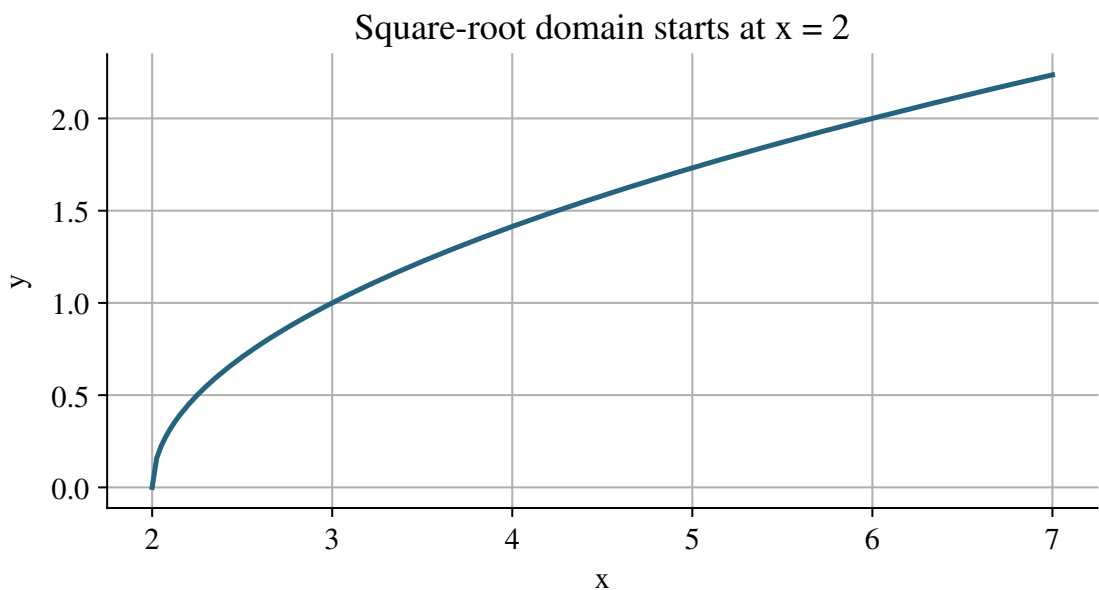
Work through a checked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $f(x) = \sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y = \sqrt{x-2}$, $x = y^2 + 2$. Thus $f^{-1}(x) = x^2 + 2$ with $x \geq 0$. For $g(x) = x+3$, $fg(1) = f(4) = \sqrt{2}$.



Test a tempting shortcut

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.
-

Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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Work through a checked case

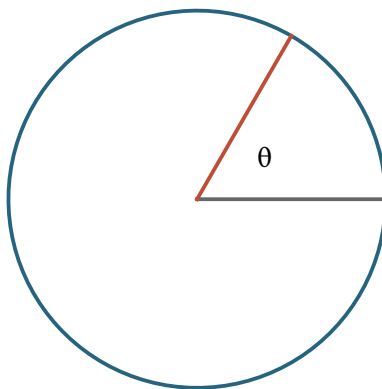
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x = 0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

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Key idea

The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.

- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

Choose the mathematical structure

- For $y=Ae^{kt}$, k is a proportional rate. Taking logs gives $\ln y = \ln A + kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.
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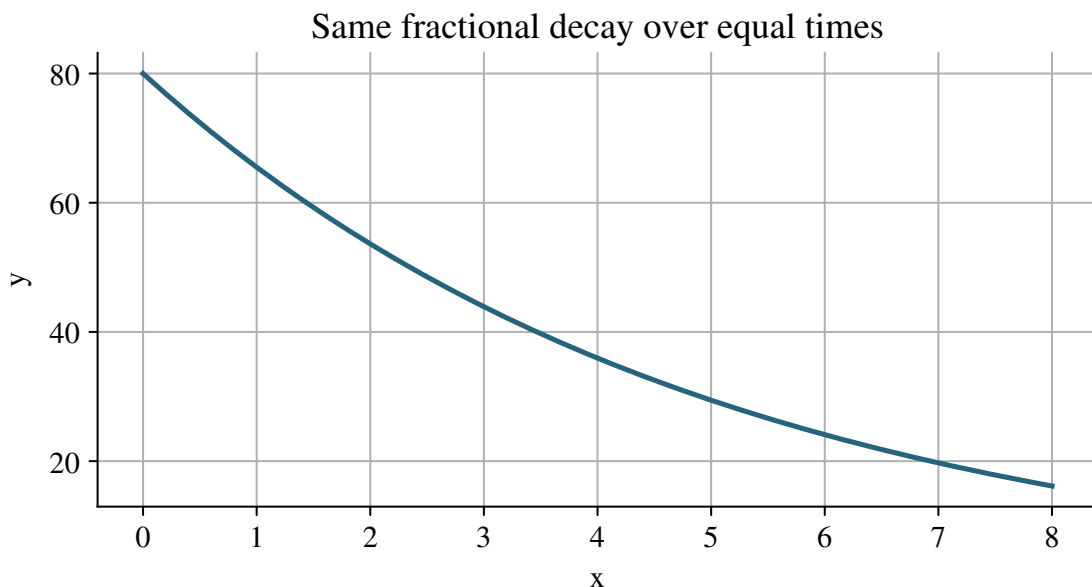
Work through a checked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

If $y=80e^{(-0.2t)}$, $y=40$ gives $e^{(-0.2t)}=1/2$. Hence $t=\ln 2/0.2 \approx 3.466$. For $3^x=20$, $x=\ln 20/\ln 3$. Plotting $\ln y$ against t linearises this exponential model.



Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For positive x and y , $\ln(x+y) = \ln x + \ln y$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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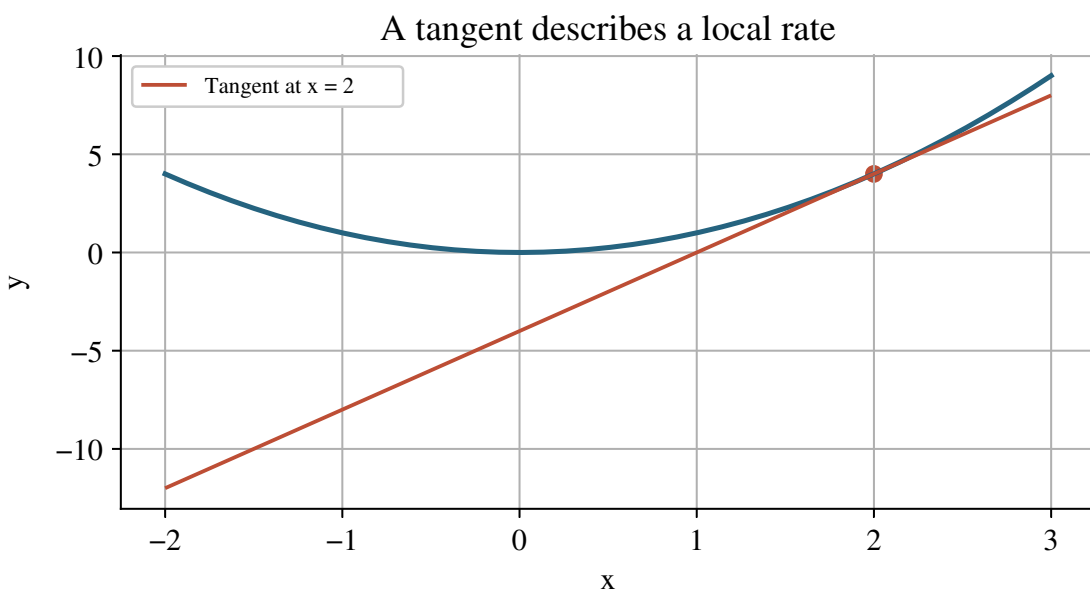
Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

Choose the mathematical structure

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$. A definite integral is signed accumulation. Split at sign changes when total area or distance is required.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

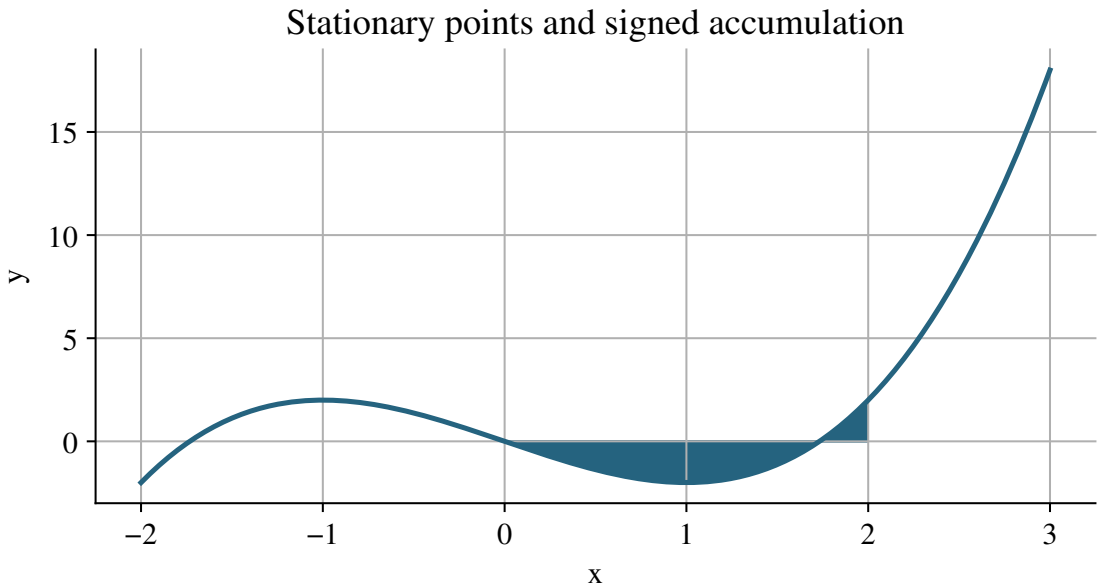
Work through a checked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $v(t)=3t^2-3$ over $0 \leq t \leq 2$, displacement $= [t^3-3t]_0^2=2$. Since v changes sign at $t=1$, distance $= -[t^3-3t]_0^1 + [t^3-3t]_1^2 = 2+4=6$. The constants cancel only for a definite integral.



Test a tempting shortcut

- The integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$. Signed area can be zero even when the total enclosed area is positive.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

P3 · Fixed-point iteration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.
-

Choose the mathematical structure

- Rearrange the equation as $x=g(x)$, choose a starting value and iterate. A limit L must satisfy $L=g(L)$. Near a fixed point, $|g'(L)| < 1$ provides a local convergence check; a sign bracket can validate the reported rounded root.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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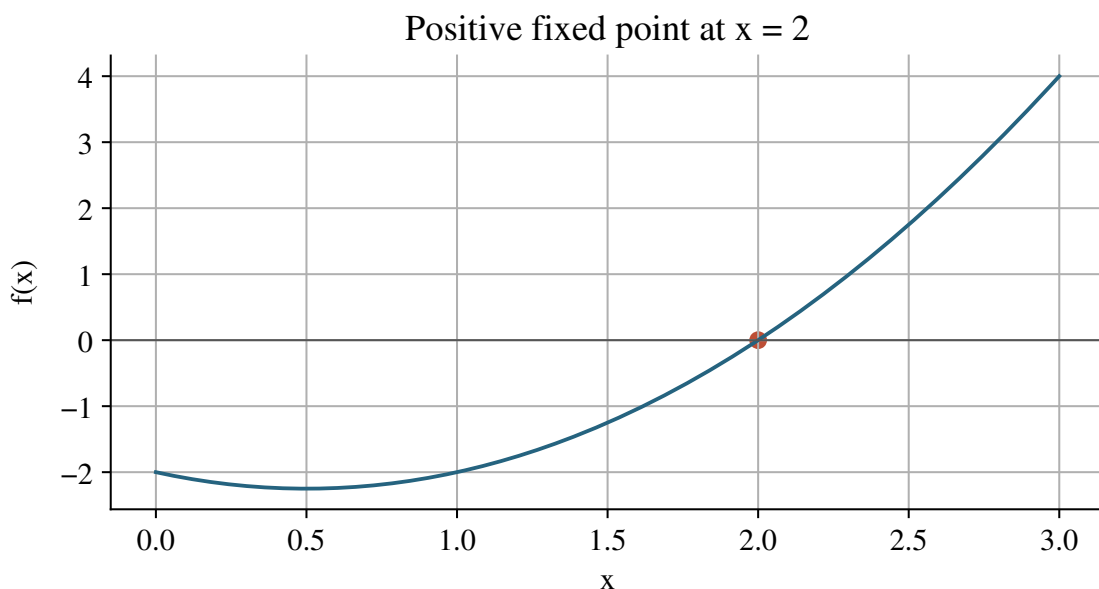
Work through a checked case

$$x_{n+1} = \sqrt{2 + x_n}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $x^2 - x - 2 = 0$ choose $x_{\text{next}} = \sqrt{2 + x}$, starting at $x_0 = 1$. Then $x_1 = \sqrt{3} \approx 1.73205$, $x_2 \approx 1.93185$ and the positive fixed point is 2. At 2, $g' = 1/(2\sqrt{4}) = 1/4$, so small local errors shrink. The square-root rearrangement seeks the positive root; it does not give the negative root -1.



Test a tempting shortcut

- Different rearrangements can have different convergence behaviour. Check the iterates, domain and original equation; a square-root update cannot reach a negative fixed point.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every rearrangement of an equation converges to the same root from every starting value. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- P3 uses equation rearrangement and numerical iteration. This lesson excludes Newton and numerical integration, which belong to other units.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
 - This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.
-

Choose the mathematical structure

- A deductive proof starts from stated definitions or assumptions. A counterexample refutes an all-values claim. Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

Odd + odd: $(2a + 1) + (2b + 1)$

Factor 2: $2(a + b + 1)$

The bracket is an integer $\rightarrow$ the sum is even

Test a tempting shortcut

- Do not assume the conclusion while proving it. In contradiction, identify the impossible consequence and reject the initial contrary assumption. Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Checking the first ten integers proves a claim for every positive integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- AQA proof includes deduction, exhaustion and contradiction. IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction. Keep the method matched to the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A single valid case that disproves a universal claim. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Equations, identities and rearrangement

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
 - This lesson studies **identity** 恒等式: An equality that holds for every allowed value of its variable.
-

Choose the mathematical structure

- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs. Preserve equality by applying the same operation to both sides. State restrictions before dividing by a variable.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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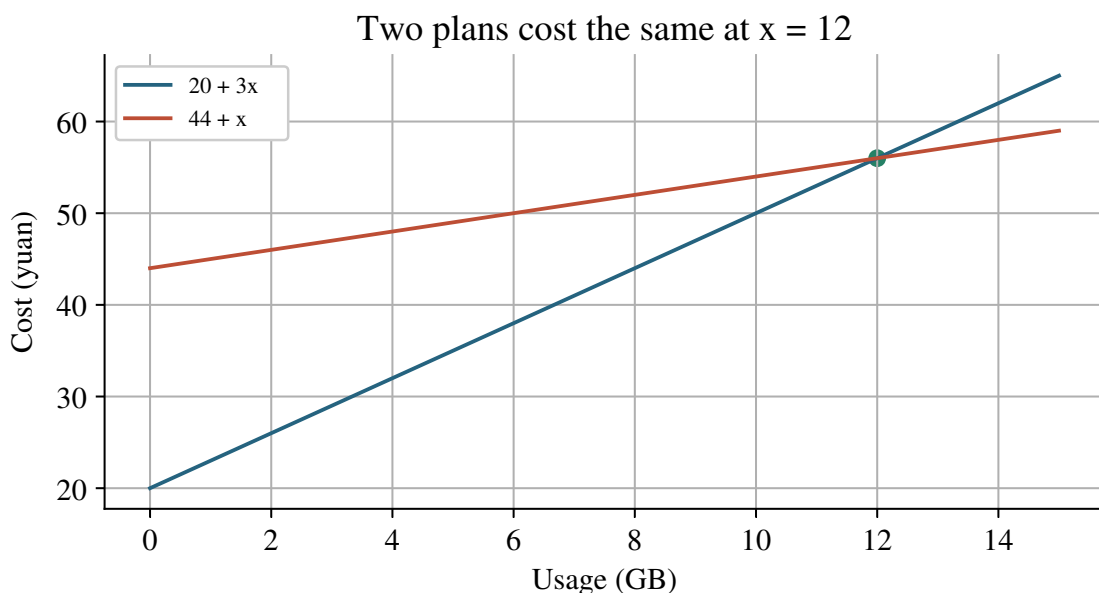
Work through a checked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{A/\pi}$, because r is a length.



Test a tempting shortcut

- Cancelling a term is not the same as cancelling a factor. In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$. Check a rearrangement by substitution.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Set up the equation from units and the meaning of the unknown. A negative or fractional solution may be algebraically correct but impossible for a count.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An equality that holds for every allowed value of its variable. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Binomial expansion and valid approximations

How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** 二项式系数: The number of ways to choose a specified number of objects from a set.

Choose the mathematical structure

- For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+\frac{n(n-1)}{2}x^2+\dots$ with $|x|<1$. Factor out constants before using this form.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

$(1+2x)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$. For $(1+x)^{-1}$, the first three terms are $1 - x + x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1 \approx 0.909091$.

$$(1 + 2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

Test a tempting shortcut

- An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$. Positive-integer expansions are finite and do not have that convergence restriction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.

Choose the mathematical structure

- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$. For parametric curves, $dy/dx = (dy/dt)/(dx/dt)$, where $dx/dt \neq 0$. A zero denominator may signal a vertical tangent.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

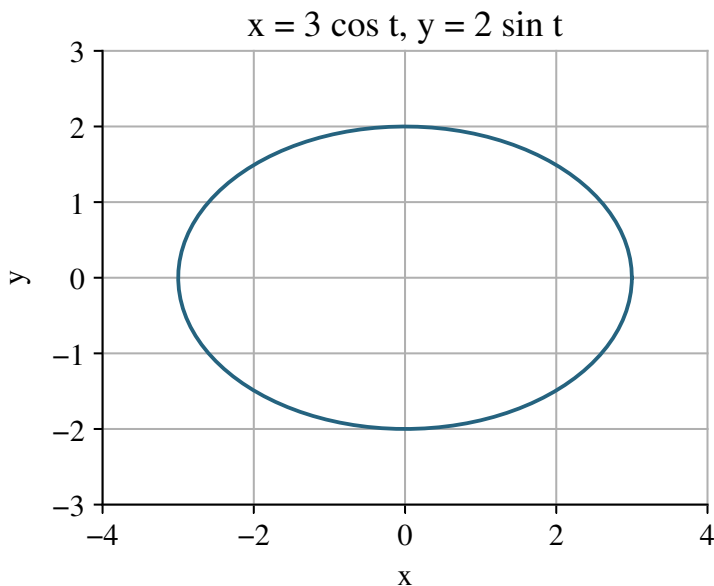
Work through a checked case

$$x = a \cos t, \quad y = b \sin t, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $x=3\cos t, y=2\sin t$ at $t=\pi/4$, the gradient is $(2\cos t)/(-3\sin t)=-2/3$. The point is $(3/\sqrt{2}, \sqrt{2})$. The area inside the ellipse is $\pi ab=6\pi$; its semi-axes are 3 and 2.



Test a tempting shortcut

- Eliminating a parameter may lose a domain restriction or direction of travel. A vertical tangent cannot be assigned a finite dy/dx . Distinguish semi-axes from full widths.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Eliminating a parameter always preserves every restriction and direction automatically. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For advanced coordinate geometry, use the defining equation and a consistent parameter. For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A representation in which coordinates are expressed using a common parameter. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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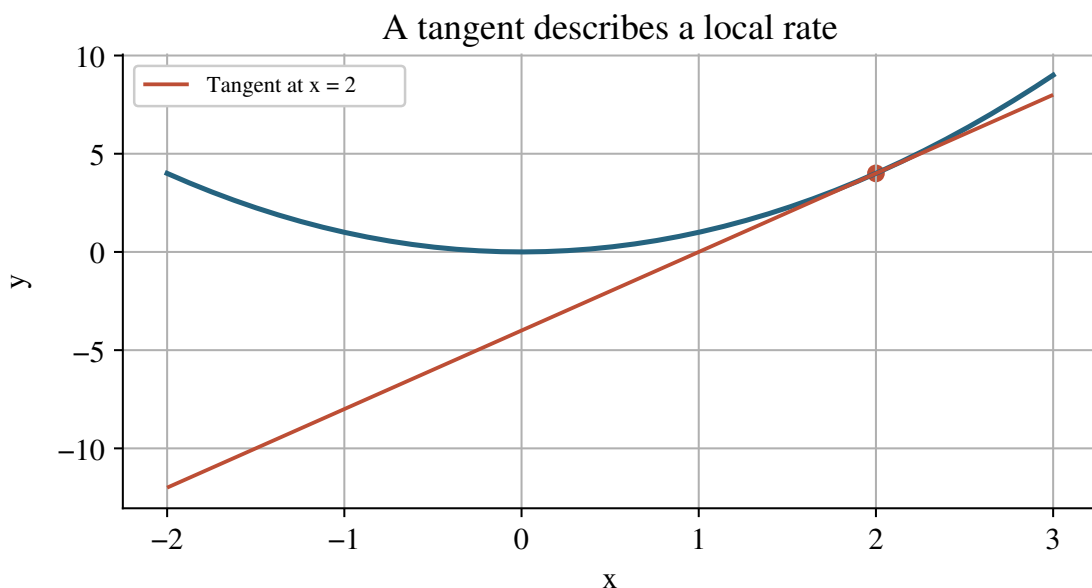
Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
 - This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.
-

Choose the mathematical structure

- Use substitution when an inner derivative appears as a factor. By parts, $\int u v' = uv - \int u' v$. For a rational function, divide first if needed, then decompose into partial fractions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

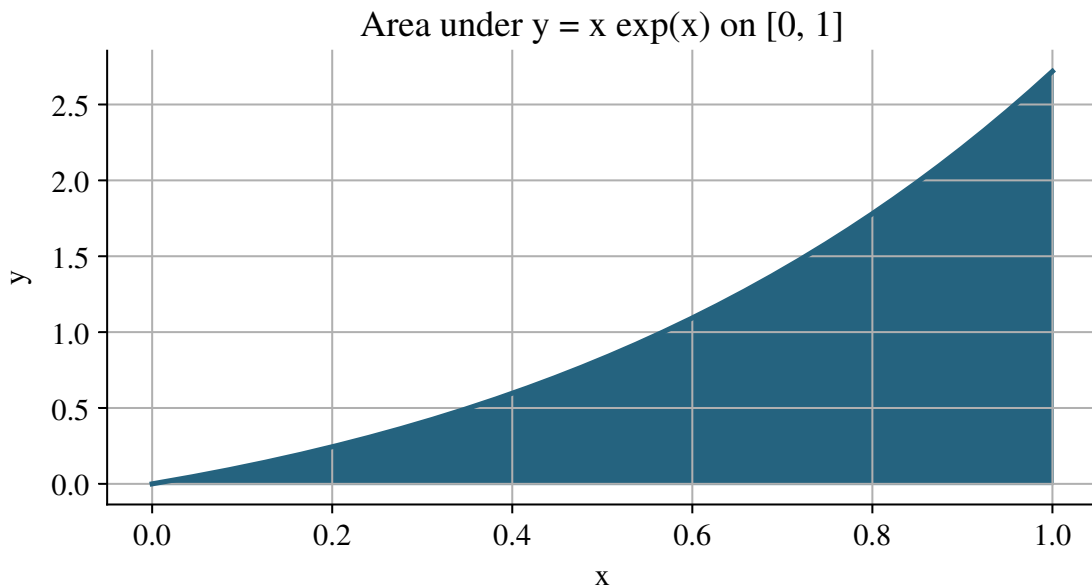
Work through a checked case

$$\int uv' dx = uv - \int u'v dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$. For integral $\int \frac{2x}{x^2+1} dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1) + C$.



Test a tempting shortcut

- Choosing u and v prime well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to x before applying the original limits.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check by differentiating. For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.

- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

Choose the mathematical structure

- Separate the variables in $dy/dx=ky$, integrate both sides and apply an initial condition. Include any equilibrium solution excluded when dividing by y .
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

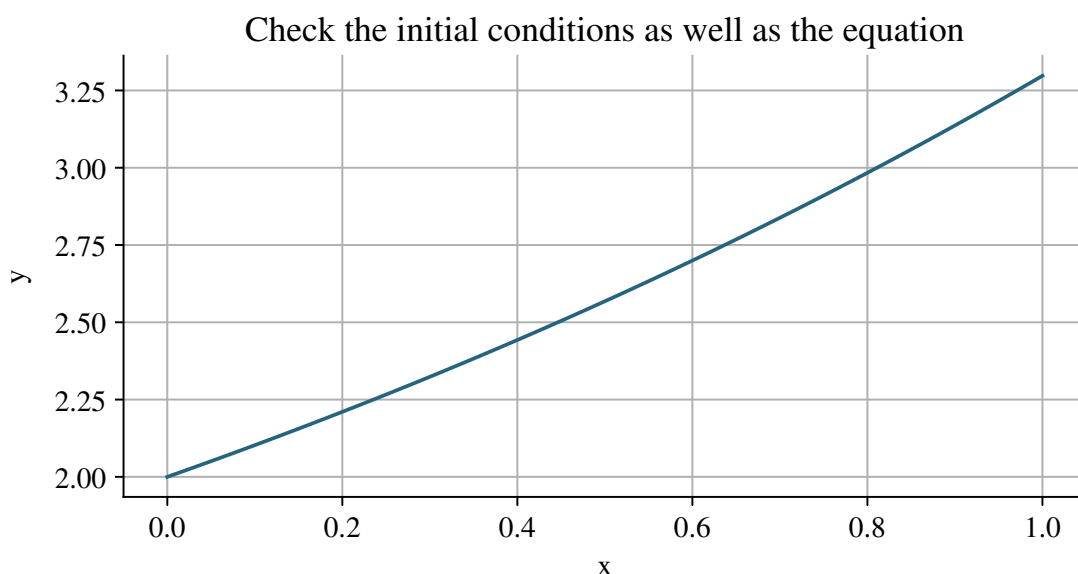
Work through a checked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $dy/dx=0.5y$ and $y(0)=2$, separation gives $\ln|y|=0.5x+C$, hence $y=Ae^{(0.5x)}$. The initial value gives $A=2$. Differentiate the result to check the equation and substitute $x=0$ to check the initial condition.



Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

An initial condition never changes the constant in a differential-equation solution. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- This unit uses first-order separation and initial conditions. Euler numerical integration and second-order complementary functions are excluded from this lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

P4 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
 - This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.
-

Choose the mathematical structure

- A line has $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ with position vector $\mathbf{a}$ and nonzero direction $\mathbf{b}$. To find an intersection, equate all components and solve for both line parameters. For an angle use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where scalar products are in the named syllabus.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

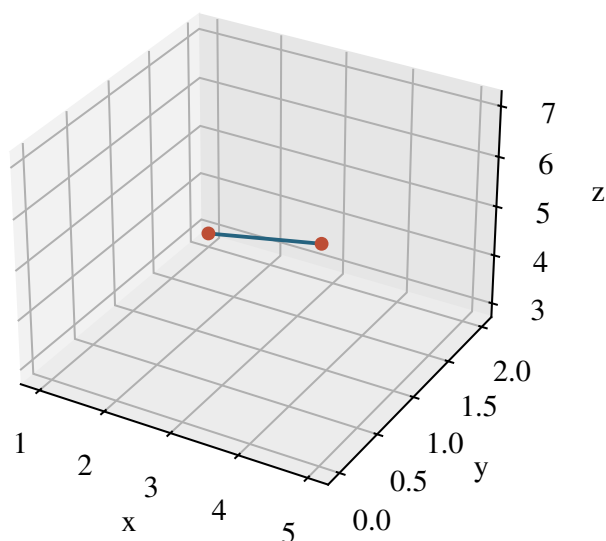
$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $\mathbf{a} = (1, 2, 3)$, $\mathbf{b} = (2, -1, 2)$, $\lambda = 2$ gives $\mathbf{r} = (5, 0, 7)$. The magnitude of $\mathbf{b}$ is $\sqrt{4+1+4}=3$. With $\mathbf{c} = (1, 0, -1)$, $\mathbf{b} \cdot \mathbf{c} = 2-2=0$, so $\mathbf{b}$ and $\mathbf{c}$ are perpendicular.

$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$



Test a tempting shortcut

- Satisfying two component equations does not guarantee the third. Nonparallel lines may be skew. A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Any two nonparallel lines in three dimensions must intersect. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State whether a requested angle is acute, directed or an angle between lines. Use the syllabus-specific plane or distance method only when it is actually required for that unit.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL pure mathematics; official unit P4. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A nonzero vector giving the direction of a line. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Complex arithmetic and quadratic roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.

Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. Add components, multiply brackets and use a conjugate to make a division denominator real. Non-real roots of a real-coefficient quadratic occur in conjugate pairs.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

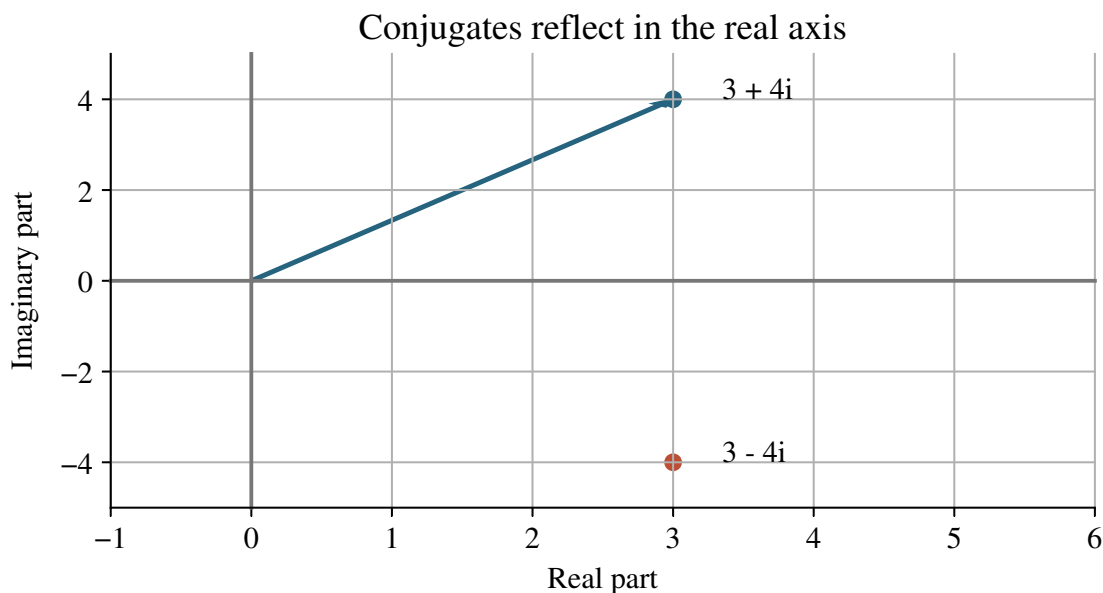
Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 uses complex arithmetic, modulus, Argand representation and quadratic roots. De Moivre powers and roots of unity are reserved for FP2.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Use this in your course

- edexcel IAL pure mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Parabolas and rectangular hyperbolas

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
- This lesson studies **parabola** 抛物线: The locus of points equally distant from a fixed focus and a fixed directrix.

Choose the mathematical structure

- For $y^2=4ax$, the focus is $(a,0)$, directrix $x=-a$ and vertex $(0,0)$. For $xy=c^2$, the coordinate axes are asymptotes. Read the parameter from the coefficient rather than assuming it equals $4a$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

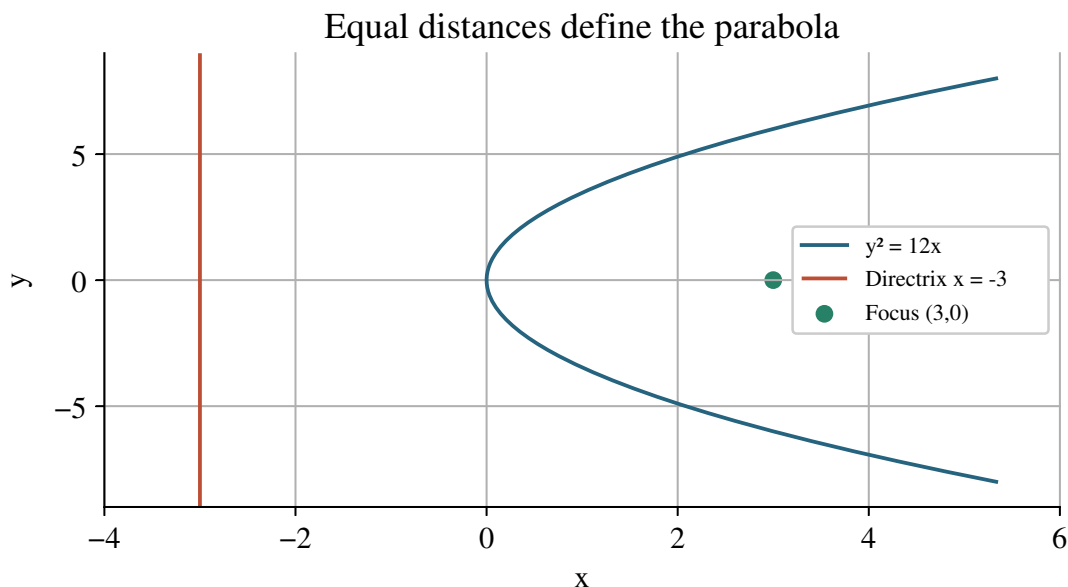
Work through a checked case

$$y^2 = 12x, \quad xy = 9$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y^2=12x$, $a=3$: the focus is $(3,0)$ and directrix $x=-3$. At $x=3$, $y=\pm 6$; at $(3,6)$, distance to the focus and directrix is 6. For $xy=9$, the point $(3,3)$ lies on the hyperbola and $x=0, y=0$ are its asymptotes.



Test a tempting shortcut

- The coefficient of x is $4a$, not a . Include both signs when solving y^2 . An asymptote is approached; it is not a finite intercept of $xy=c^2$.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For $y^2=4ax$, the focus x -coordinate is $4a$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 coordinate-geometry support for parabolas and rectangular hyperbolas. Ellipse area and ellipse parametrisation are excluded.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The locus of points equally distant from a fixed focus and a fixed directrix. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Matrix operations, inverses and plane transformations

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
 - This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
-

Choose the mathematical structure

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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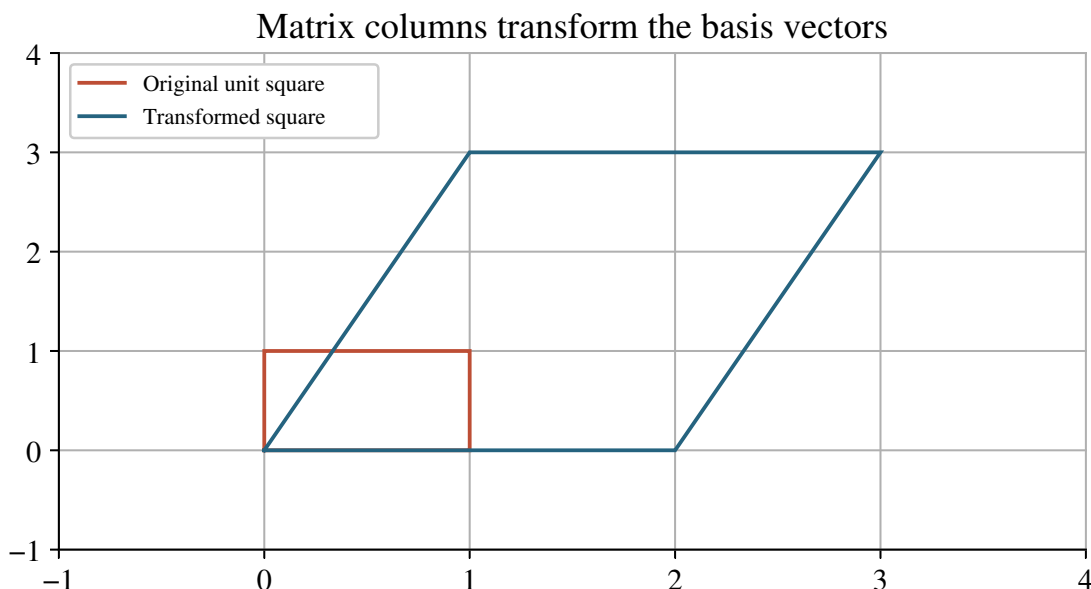
Work through a checked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. Check by multiplying A by its inverse to obtain the identity matrix.



Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 uses 2×2 matrix operations, inverses and plane transformations. Eigenvalues and 3×3 diagonalisation belong to FP3, not this unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Bisection and Newton root finding

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.
-

Choose the mathematical structure

- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs. Newton updates x to $x - f(x)/f'(x)$, provided the derivative is nonzero.

- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

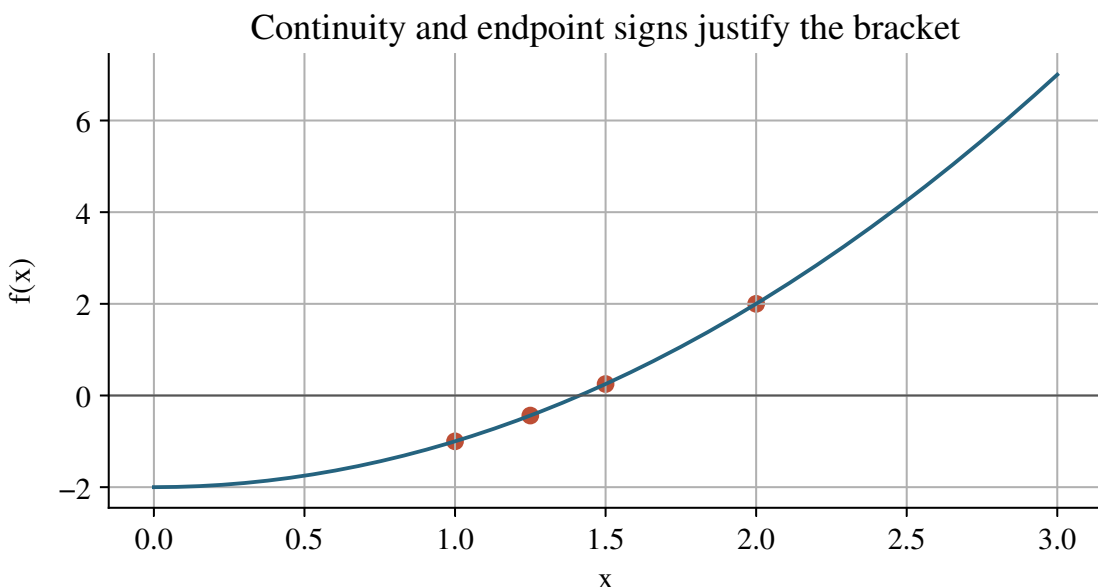
Work through a checked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $f(x)=x^2-2$, $f(1)=-1$ and $f(2)=2$. At midpoint 1.5, $f=0.25$, so the new bracket is $[1,1.5]$. At midpoint 1.25, $f=-0.4375$, so the next bracket is $[1.25,1.5]$. Newton from 1.5 gives 1.4166667.



Test a tempting shortcut

- A sign change must occur on an interval where the function is continuous. Newton can fail if its derivative is zero, or if its iterates leave the useful domain.

- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every sign change proves a root, including one across a discontinuity. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 numerical methods concern roots: bisection, interpolation and Newton methods. Numerical integration is not included in this lesson.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Finite sums and induction

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
 - This lesson studies **sigma notation** 求和符号: Notation that adds indexed terms over a stated finite range.
-

Choose the mathematical structure

- For $k=1$ to n , $\text{sum } k = n(n+1)/2$, $\text{sum } k^2 = n(n+1)(2n+1)/6$ and $\text{sum } k^3 = [n(n+1)/2]^2$. Differences between consecutive terms can telescope. In induction, verify the starting case and show the next case follows from the assumed case.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

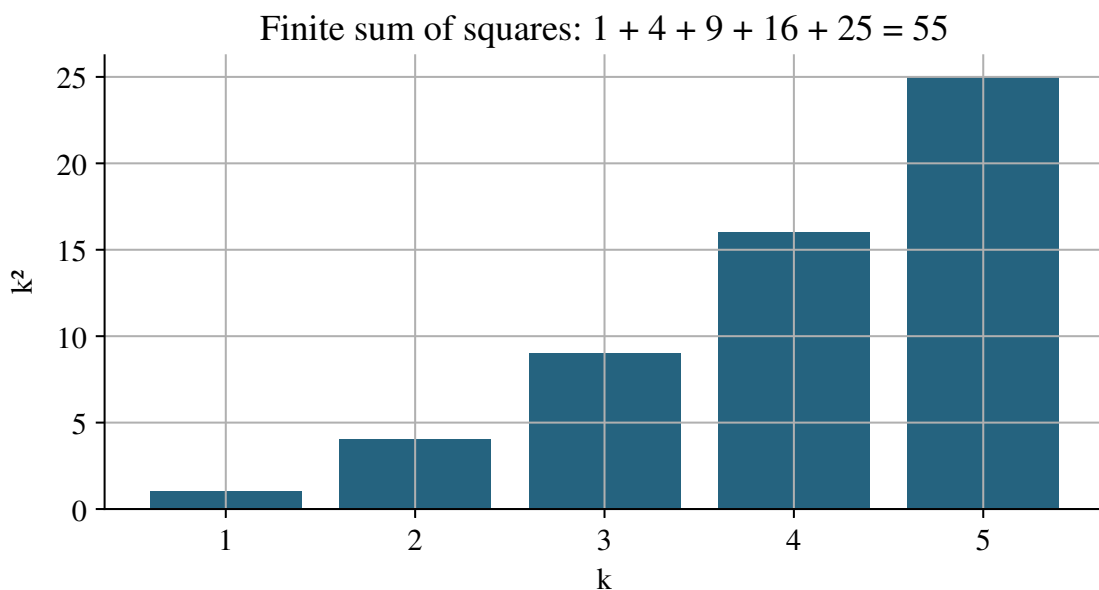
Work through a checked case

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $n=5$, $\text{sum } k=15$, $\text{sum } k^2=1+4+9+16+25=55$ and $\text{sum } k^3=1+8+27+64+125=225$. In proving a sum formula, adding the $(n+1)$ th term to the assumed expression must produce the formula with n replaced by $n+1$.



Test a tempting shortcut

- The sum of squares is not the square of the sum. State the index range and check the first case; an inductive step by itself is not a complete induction proof.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The sum of squared terms always equals the square of their sum. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- FP1 finite series and induction support only. Maclaurin and Taylor approximation belong to later further-pure units and are excluded.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

Notation that adds indexed terms over a stated finite range. Choose the relationship, show the method, check its assumptions and interpret the result.

FP1 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

Choose the mathematical structure

- A deductive proof starts from stated definitions or assumptions. A counterexample refutes an all-values claim. Induction requires a base case and an implication from $n=k$ to $n=k+1$; checking successive cases alone is not induction.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

$$(2a + 1) + (2b + 1) = 2(a + b + 1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For odd integers $2a+1$ and $2b+1$, their sum is $2(a+b+1)$, hence even. The claim n^2+n+41 is always prime fails at $n=41$: the value is $41 \times 43 = 1763$. For $1+\dots+n=n(n+1)/2$, the induction step adds $k+1$ to the assumed sum.

Odd + odd: $(2a + 1) + (2b + 1)$

Factor 2: $2(a + b + 1)$

The bracket is an integer $\rightarrow$ the sum is even

Test a tempting shortcut

- Do not assume the conclusion while proving it. In contradiction, identify the impossible consequence and reject the initial contrary assumption. Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Checking the first ten integers proves a claim for every positive integer. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- AQA proof includes deduction, exhaustion and contradiction. IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction. Keep the method matched to the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP1. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A single valid case that disproves a universal claim. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
 - This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
-

Choose the mathematical structure

- Factor where possible; otherwise complete the square or use the quadratic formula. A quadratic inequality needs the sign on intervals, not only the roots. The discriminant identifies repeated or missing real roots.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

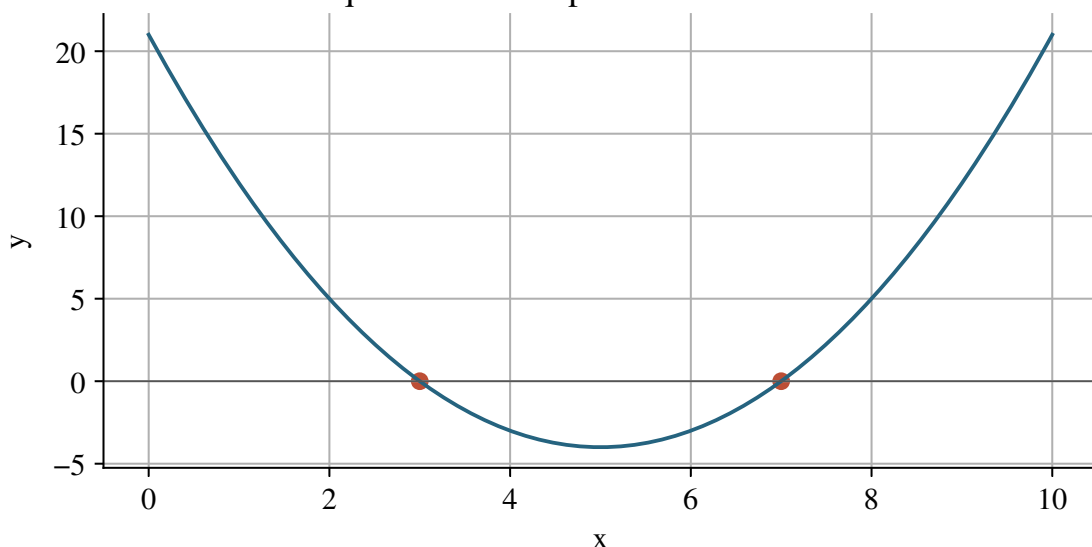
$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The condition is $x(10-x) \geq 21$, so $x^2-10x+21 \leq 0$. Factor $(x-3)(x-7) \leq 0$. The upward parabola is nonpositive between its roots, giving $3 \leq x \leq 7$. The maximum area is 25 at $x=5$.

The quadratic is nonpositive between 3 and 7



Test a tempting shortcut

- Multiplying an inequality by a negative number reverses its direction. A sketch must show which side of each root satisfies the inequality. Geometry may restrict x further.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A positive discriminant means that a quadratic has no real roots. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use the vertex to interpret an optimum. Check an endpoint and a point between the roots; the algebra and graph should tell the same story.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Series, finite differences and Taylor expansions

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **Taylor polynomial** 泰勒多项式: A polynomial formed from a function's derivatives at a chosen centre.

Choose the mathematical structure

- About $x=a$, $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots$ when the expansion is valid. At $a=0$ it is a Maclaurin expansion. Finite sums may also simplify by cancellation or standard sum formulae.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

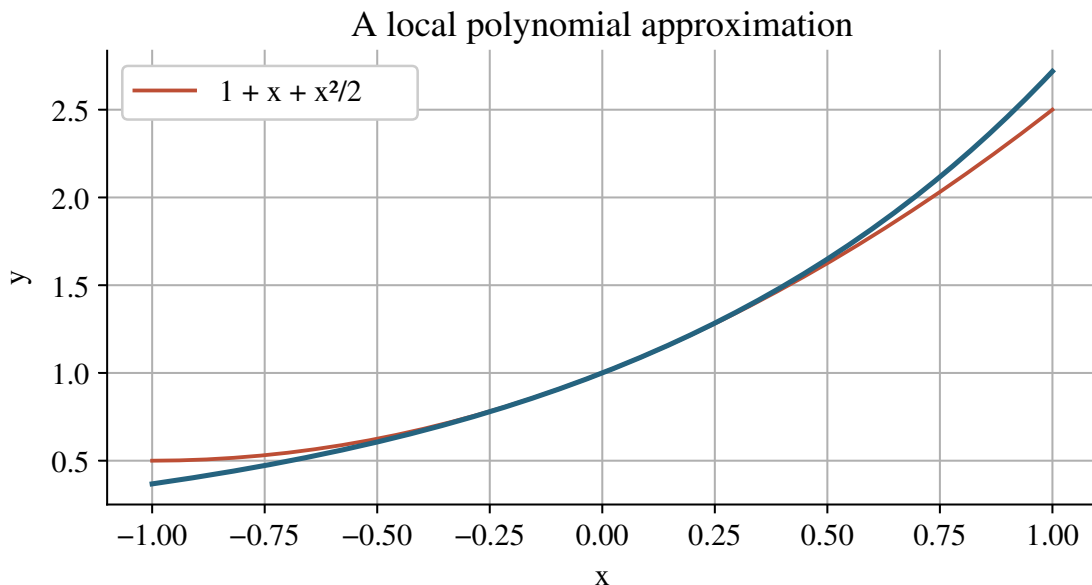
Work through a checked case

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

The first three terms of e^x are $1+x+x^2/2$. At $x=0.1$ this gives 1.105, close to $e^{0.1} \approx 1.105170$. For the sum of k^2 from $k=1$ to 5, $n(n+1)(2n+1)/6$ gives $5 \times 6 \times 11/6 = 55$.



Test a tempting shortcut

- The factorial belongs in the denominator of every Taylor coefficient. The expansion centre is not always zero. An approximation close to its centre can be poor far away.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

A Taylor polynomial is always exact for the original function at every input. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For a telescoping series, write several terms and identify both surviving ends. For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A polynomial formed from a function's derivatives at a chosen centre. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Complex numbers and roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
 - This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.
-

Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate $=a^2+b^2$. The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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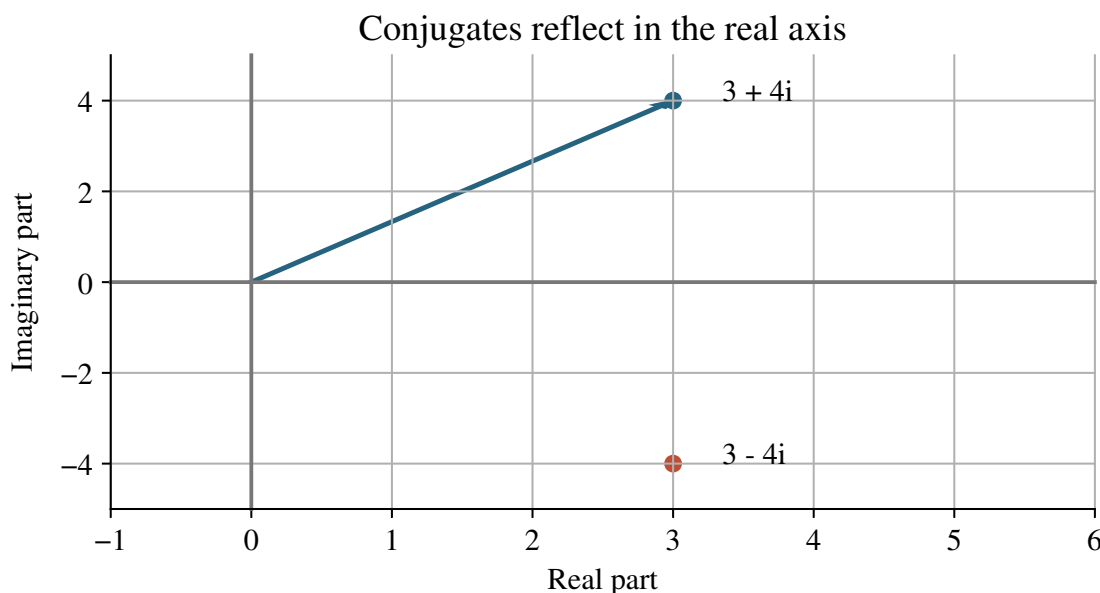
Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
 - This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.
-

Choose the mathematical structure

- For $dy/dx=ky$, separate variables: $dy/y=k dx$, giving $y=Ae^{(kx)}$. Use an initial condition to find A. Euler's method takes $y_{next}=y+h f(x,y)$, with a chosen step h .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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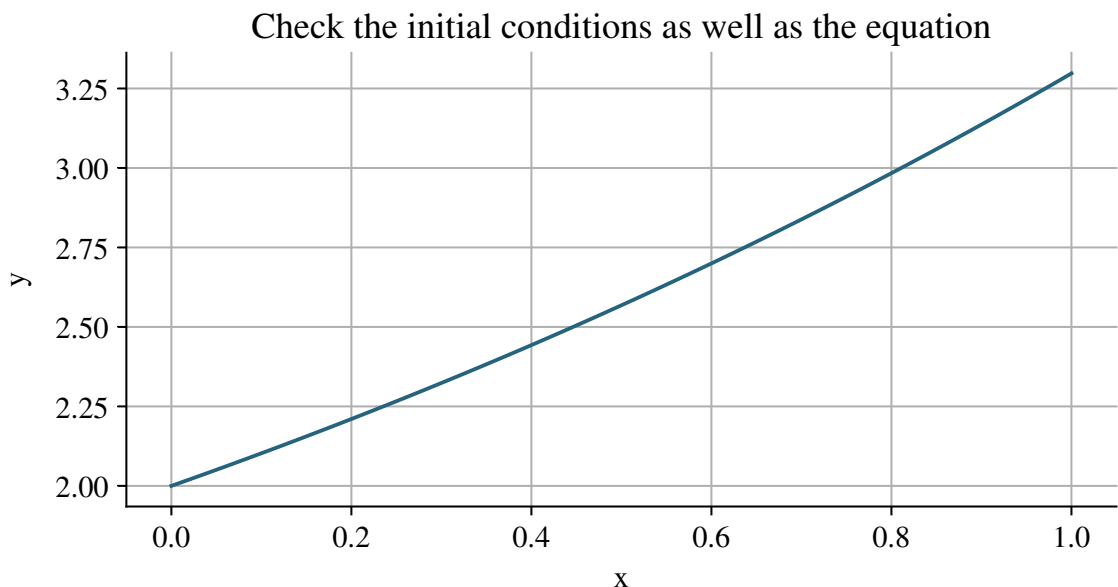
Work through a checked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

With $\frac{dy}{dx}=0.5y$ and $y(0)=2$, $y=2e^{(0.5x)}$. Euler with $h=0.2$ gives $y(0.2)\approx 2+0.2\times 1=2.2$ and $y(0.4)\approx 2.2+0.2\times 1.1=2.42$. The exact second value is about 2.4428.



Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every step size gives exactly the same Euler approximation. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions. This is Further Pure scope; check the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Second-order linear differential equations

Why do we need two initial conditions?

- A displacement model involves both velocity and acceleration. Solving only a first-order rate equation cannot capture both initial conditions.
 - This lesson studies **complementary function** 互补函数: The general solution of the associated homogeneous linear differential equation.
-

Choose the mathematical structure

- For $y'' + ay' + by = f(x)$, solve the auxiliary quadratic for the homogeneous part. Add a suitable particular integral for the forcing term. Repeated and complex roots require their correct forms.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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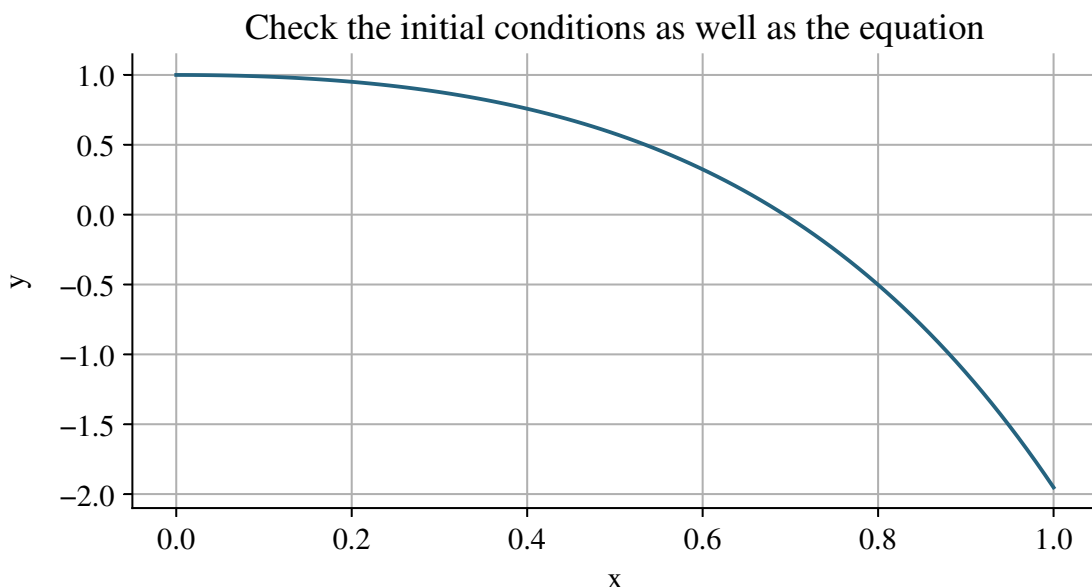
Work through a checked case

$$y'' + ay' + by = f(x), \quad m^2 + am + b = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y'' - 3y' + 2y = 0$, the auxiliary equation is $m^2 - 3m + 2 = 0$ with roots 1, 2. Hence $y = Ae^x + Be^{2x}$. With $y(0) = 1$ and $y'(0) = 0$, $A + B = 1$ and $A + 2B = 0$, so $A = 2, B = -1$.



Test a tempting shortcut

- Two arbitrary constants need two independent conditions. If the trial particular integral duplicates a complementary-function term, multiply the trial by x as required. Do not discard a valid oscillatory solution because the auxiliary roots are complex.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

One initial value is always sufficient to determine both constants in a second-order general solution. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Differentiate the final expression and substitute into the original differential equation. Check both initial conditions separately, and interpret the permitted solution interval.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The general solution of the associated homogeneous linear differential equation. Choose the relationship, show the method, check its assumptions and interpret the result.

FP2 · Polar curves and area

Why describe a curve by direction?

- A petal-shaped curve is simpler when its radius depends on direction. Cartesian coordinates can hide this structure.
 - This lesson studies **polar coordinate** 极坐标: A position described by distance r and angle θ from a chosen origin and reference ray.
-

Choose the mathematical structure

- Use $x=r \cos \theta$ and $y=r \sin \theta$. A polar area is one half the integral of r^2 with respect to θ over a correctly chosen interval. Identify symmetry and repeated tracing before selecting limits.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

Work through a checked case

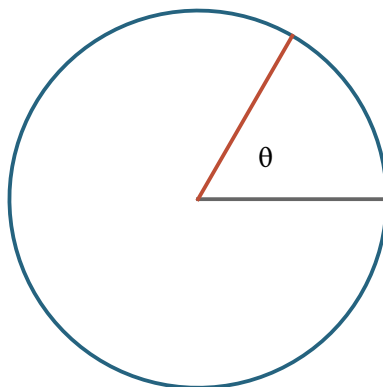
$$x = r \cos \theta, \quad y = r \sin \theta, \quad A = \frac{1}{2} \int r^2 d\theta$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $r=2$ and $\theta=\pi/3$, $x=1$ and $y=\sqrt{3}$. For the circle $r=2$ over a complete turn, $A=(1/2) \int_0^{2\pi} 4 d\theta=4\pi$. A half-turn gives 2π , exactly half the disk.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

Test a tempting shortcut

- Negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray. A parametrisation may trace the same region more than once, so a full parameter interval can overcount area.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Using any full parameter interval always counts each polar region exactly once. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Sketch enough points to establish orientation and bounds. For an enclosed region between two curves, determine intersections and which radial square contributes the outer boundary on each interval.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP2. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A position described by distance r and angle θ from a chosen origin and reference ray. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Hyperbolic functions and inverse relations

How can growth and decay make a symmetric curve?

- A hanging cable has a curved profile related to exponentials. Hyperbolic functions combine growth and decay symmetrically.
 - This lesson studies **hyperbolic cosine** 双曲余弦: The function $\cosh x = (e^x + e^{-x})/2$.
-

Choose the mathematical structure

- Define $\sinh x = (e^x - e^{-x})/2$ and $\cosh x = (e^x + e^{-x})/2$. Their identity is $\cosh^2 x - \sinh^2 x = 1$. Derivatives are $\sinh' = \cosh$ and $\cosh' = \sinh$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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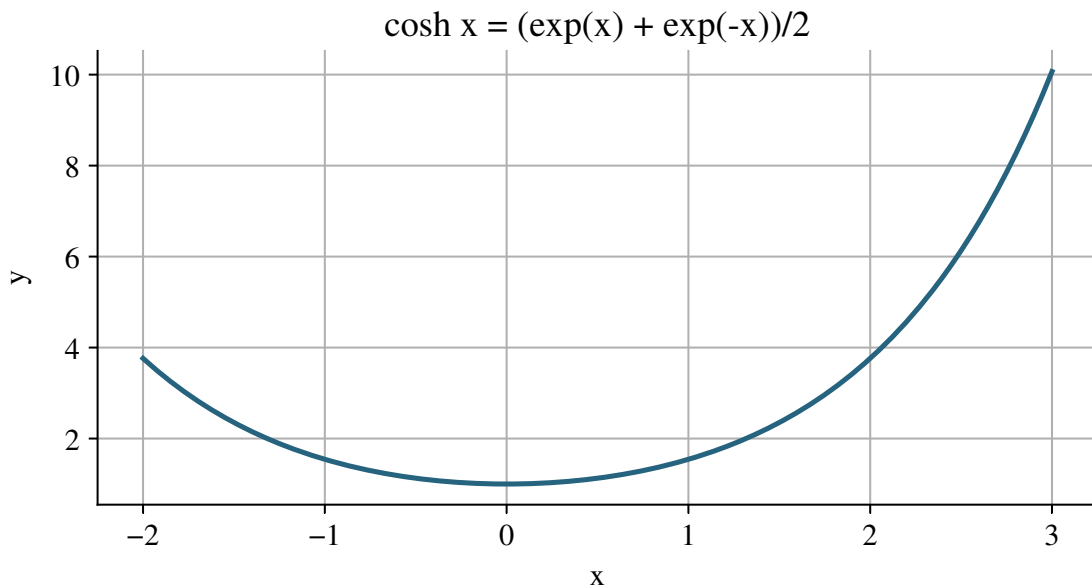
Work through a checked case

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \cosh^2 x - \sinh^2 x = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

At $x=\ln 2$, $e^x=2$ and $e^{-x}=1/2$. Thus $\cosh x=1.25$ and $\sinh x=0.75$. Their squared difference is $1.5625-0.5625=1$. To invert $y=\sinh x$, solve a quadratic in e^x and choose the positive root.



Test a tempting shortcut

- The hyperbolic identity has a minus sign. $\cosh$ is not one-to-one on all real inputs; its usual inverse uses $x \geq 0$. Ordinary circular-trigonometric identities cannot be substituted unchanged.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The identity for hyperbolic functions is $\cosh^2 x + \sinh^2 x = 1$. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Use exponential definitions to prove identities and solve equations. State domain restrictions for inverse functions before differentiating or integrating them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The function $\cosh x = (e^x + e^{-x})/2$. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single $y=f(x)$.
 - This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.
-

Choose the mathematical structure

- For $x=a \cos t, y=b \sin t$, eliminating t gives $x^2/a^2 + y^2/b^2 = 1$. For parametric curves, $dy/dx = (dy/dt)/(dx/dt)$, where $dx/dt \neq 0$. A zero denominator may signal a vertical tangent.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

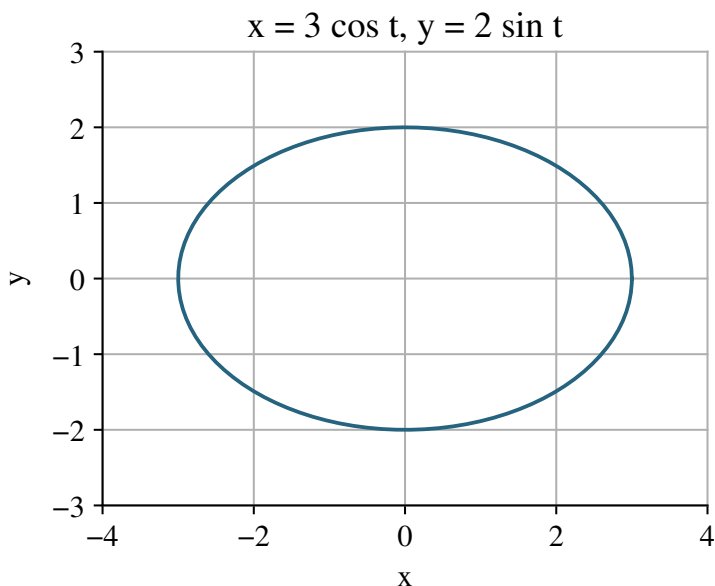
Work through a checked case

$$x = a \cos t, \quad y = b \sin t, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $x=3\cos t, y=2\sin t$ at $t=\pi/4$, the gradient is $(2\cos t)/(-3\sin t)=-2/3$. The point is $(3/\sqrt{2}, \sqrt{2})$. The area inside the ellipse is $\pi ab=6\pi$; its semiaxes are 3 and 2.



Test a tempting shortcut

- Eliminating a parameter may lose a domain restriction or direction of travel. A vertical tangent cannot be assigned a finite dy/dx . Distinguish semiaxes from full widths.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Eliminating a parameter always preserves every restriction and direction automatically. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- For advanced coordinate geometry, use the defining equation and a consistent parameter. For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A representation in which coordinates are expressed using a common parameter. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

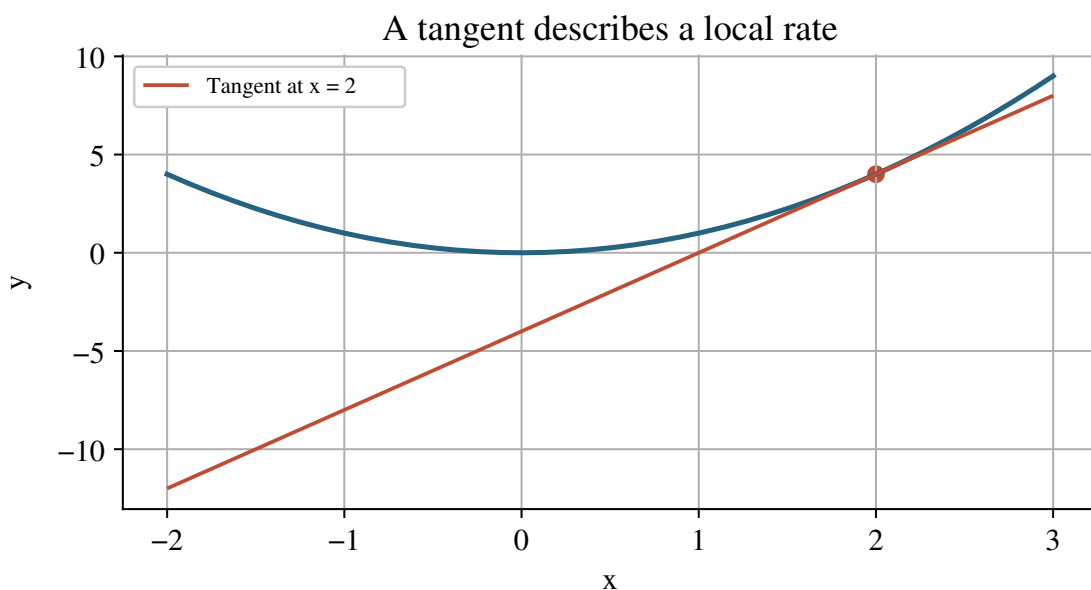
Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as $x e^x$ cannot be integrated by separately integrating each factor.
 - This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.
-

Choose the mathematical structure

- Use substitution when an inner derivative appears as a factor. By parts, $\int u v' = uv - \int u' v$. For a rational function, divide first if needed, then decompose into partial fractions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

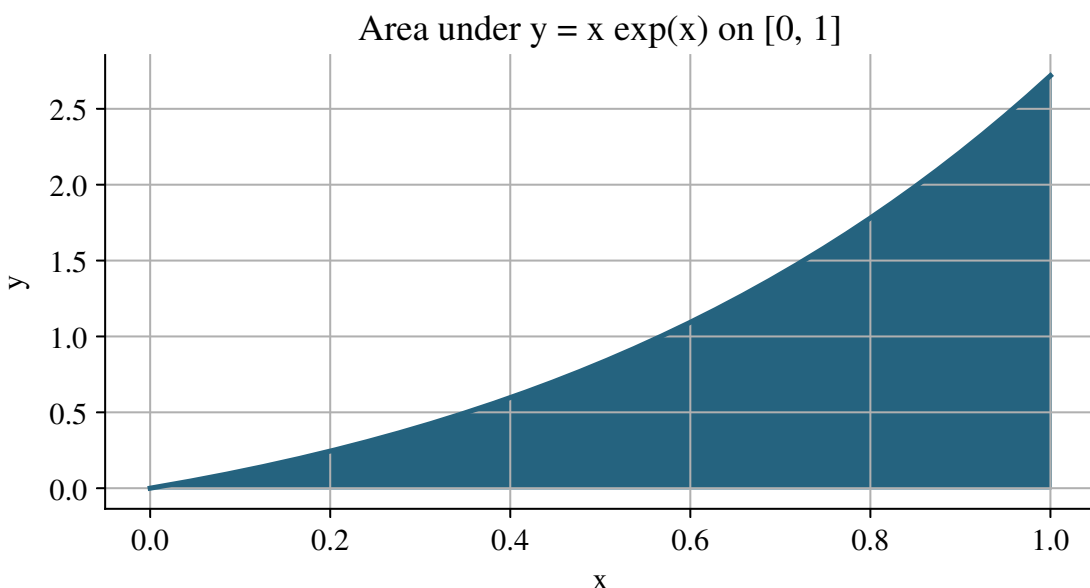
Work through a checked case

$$\int u v' dx = uv - \int u' v dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$.
For integral $\int \frac{2x}{x^2+1} dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1) + C$.



Test a tempting shortcut

- Choosing u and v prime well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to x before applying the original limits.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Check by differentiating. For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.

Choose the mathematical structure

- A line has $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ with position vector $\mathbf{a}$ and nonzero direction $\mathbf{b}$. To find an intersection, equate all components and solve for both line parameters. For an angle use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where scalar products are in the named syllabus.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Work through a checked case

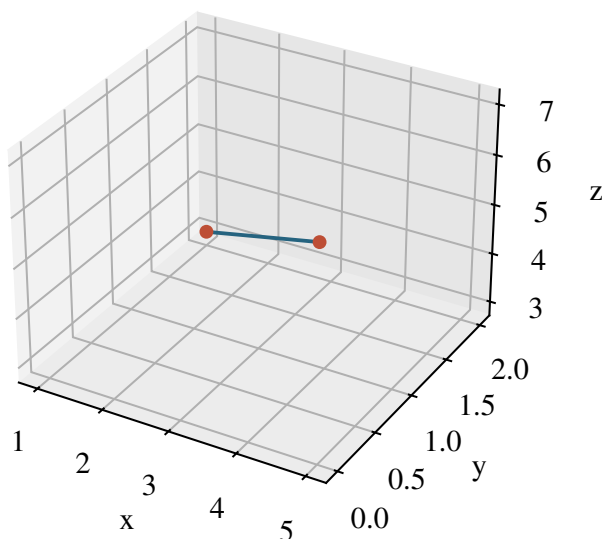
$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $a=(1,2,3), b=(2,-1,2)$, $\lambda=2$ gives $r=(5,0,7)$. The magnitude of b is $\sqrt{(4+1+4)}=3$. With $c=(1,0,-1)$, $b \cdot c=2-2=0$, so b and c are perpendicular.

$$r = (1, 2, 3) + \lambda(2, -1, 2)$$



Test a tempting shortcut

- Satisfying two component equations does not guarantee the third. Nonparallel lines may be skew. A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Any two nonparallel lines in three dimensions must intersect. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- State whether a requested angle is acute, directed or an angle between lines. Use the syllabus-specific plane or distance method only when it is actually required for that unit.

- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A nonzero vector giving the direction of a line. Choose the relationship, show the method, check its assumptions and interpret the result.

FP3 · Matrix transformations, inverses and eigenvalues

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
 - This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
-

Choose the mathematical structure

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

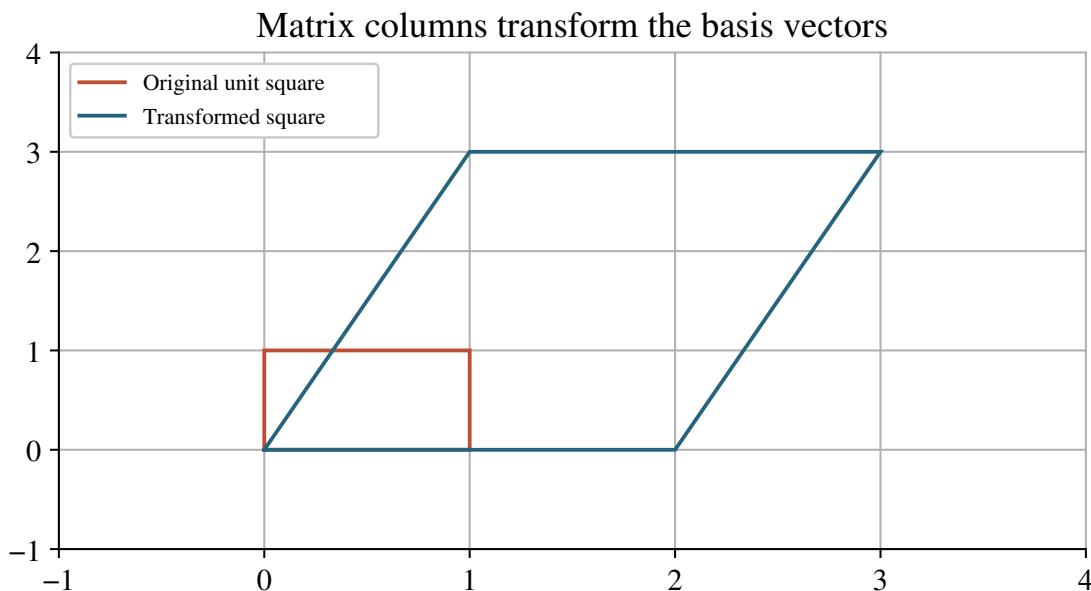
Work through a checked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Worked example

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.



Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Watch out

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

Interpret a new situation

- Find an eigenvector by solving $(A - \lambda I)v = 0$ with $v \neq 0$. Explain the geometrical meaning: this vector keeps its line direction under the transformation.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

Use this in your course

- edexcel IAL pure mathematics; official unit FP3. Other-unit enrichment is identified in the scope review; it is not an extra cash-in requirement.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key idea

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions and interpret the result.