

Unit 4: Further Mechanics, Fields and Particles

WPH14 · Issue 3 · requirements 81–124 · sheets 4.1–4.7

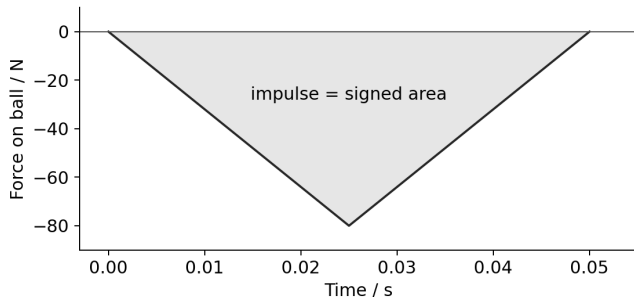
One chain: force, motion and evidence

- A bumper and a particle detector both use momentum.
- WPH14: requirements 81–124, with Units 1–2 prerequisites.
- Use the seven skill sheets before the authentic sets.

Momentum and impulse carry signs

- Momentum: $\mathbf{p} = m\mathbf{v}$.
- Impulse: $F_{\text{av}}\Delta t = \Delta p = m(v - u)$.
- Choose a positive direction before subtraction.

A pulse gives signed impulse



- Signed area gives impulse: $\Delta p = -2.0 \text{ N s}$.
- Average force is area divided by time, not peak force.

Your turn: a rebound

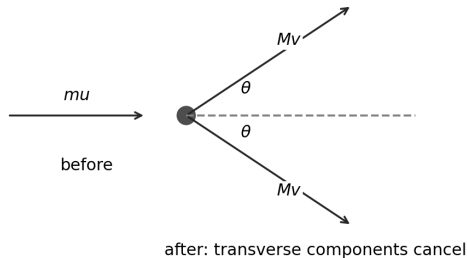
- Mass 0.30 kg; approach $+5.0$ m/s; rebound -3.0 m/s.
- Contact time 0.040 s.
- Find impulse and average force before advancing.

Rebound: use final minus initial

1 **Impulse** $\Delta p = m(v - u) = 0.30 \text{ kg} \times (-3.0 - 5.0) \text{ m/s} = -2.4 \text{ N s}$

2 **Average force** $F_{\text{av}} = \Delta p / \Delta t = (-2.4 \text{ N s}) / (0.040 \text{ s}) = -60 \text{ N}$

Conserve two components, not magnitudes



- In an isolated system, conserve momentum along each perpendicular axis.
- Compare kinetic energies to decide whether the collision is elastic.

Elasticity and the practical evidence

- Elastic collision: total kinetic energy also conserved. Inelastic collision: it decreases in a passive collision.
- For non-relativistic particles, $E_k = p^2/(2m)$; at equal E_k , $p \propto \sqrt{m}$.
- CP9: force–time area versus momentum change. CP10: calibrated overhead collision video.

CP9: compare independent measurements

- Zero a force sensor; record a complete, well-sampled force–time pulse.
- Measure trolley mass and signed velocities independently.
- Compare graph impulse with $m(v - u)$; repeat and discuss friction/offsets.

CP10: calibrate a two-dimensional collision

- Overhead camera perpendicular to the plane; scale in that plane; known frame rate.
- Track sphere centres over several frames before/after contact.
- Resolve velocities; compare momentum components and total kinetic energy with uncertainty.

Radians connect rotation to speed

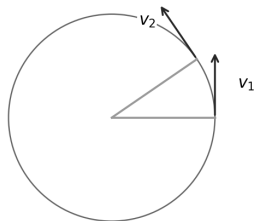
- 1 **Convert rotations** $f = N/t = 120/(60 \text{ s}) = 2.0 \text{ Hz}$
- 2 **Angular velocity** $\omega = 2\pi f = 2\pi \times 2.0 \text{ s}^{-1} = 12.6 \text{ rad/s}$
- 3 **Tangential speed** $v = \omega r = 4\pi \text{ s}^{-1} \times 0.25 \text{ m} = 3.14 \text{ m/s}$
- 4 **Acceleration** $a = r\omega^2 = 0.25 \text{ m} \times (4\pi \text{ s}^{-1})^2 = 39.5 \text{ m/s}^2$

One rigid turntable, different tangential speeds

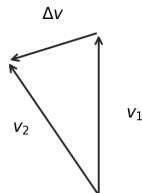
- $\theta_{\text{rad}} = \theta_{\text{degrees}}(2\pi/360)$; $\omega = \Delta\theta/\Delta t = 2\pi/T$.
- Points on one rigid turntable share angular speed.
- Since $v = \omega r$, a point at twice the radius has twice the tangential speed.

Derive the inward acceleration

Tangential velocity vectors



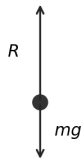
$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$



- Similar velocity and position triangles give $\Delta v/v = \Delta s/r$.
- As $\Delta t \rightarrow 0$, $a = v^2/r$ points towards the centre.

Centripetal force is the resultant

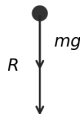
Bottom of drum



$$R - mg = mv^2/r$$

Top of drum

$$R + mg = mv^2/r$$



- Resolve actual forces towards the centre.
- The inward resultant supplies mv^2/r , do not add an extra force.

Top and bottom need different force equations

- At the bottom, inward is upward: $R - mg = mv^2/r$.
- At the top, inward is downward: $R + mg = mv^2/r$.
- If the required contact force would be negative, the assumed contact has failed.

Your turn: the friction limit

- A horizontal turntable rotates at 2.0 rad/s .
- Maximum friction is $mg/20$.
- Find the largest radius and decide whether doubling mass changes it.

Mass cancels for this friction model

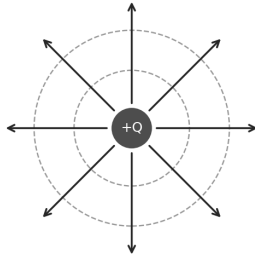
- 1 **Limit** $mr\omega^2 \leq mg/20 \Rightarrow r_{\max} = g/(20\omega^2)$
- 2 **Substitute** $r_{\max} = g/(20\omega^2) = (9.81 \text{ m/s}^2)/[20(2.0 \text{ s}^{-1})^2] = 0.123 \text{ m}$
- 3 **Interpret** Required force and maximum friction both scale with mass. Initial release motion is tangent.

Field, potential and work are different

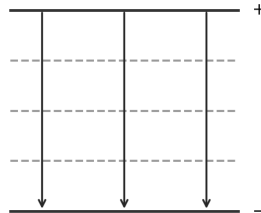
Quantity	Relationship	Meaning
Electric field strength	$\mathbf{E} = \mathbf{F}/q$	Vector
Electric potential	$\Delta U = q\Delta V$	Signed scalar
Work by field	$W_{\text{field}} = -q\Delta V$	Opposite energy change

Radial and uniform representations

Radial field and equipotentials



Uniform field between plates



- Field lines point away from a positive source charge.
- Uniform plates give $E = V/d$ away from edge effects.

One sphere, two different distances

1 Surface charge

$$Q = Vr/k = (9.0 \times 10^3 \text{ V})(0.20 \text{ m})/(8.99 \times 10^9 \text{ N m}^2/\text{C}^2) = 2.00 \times 10^{-7} \text{ C}$$

2 Outside field

$$E = kQ/r^2 = (8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(2.00 \times 10^{-7} \text{ C})/(0.30 \text{ m})^2 = 2.00 \times 10^4 \text{ N/C}$$

Potential comes from the correct area

- Radial direction: $E_r = -dV/dr$.
- With zero at infinity: $V(r) = \int_r^\infty E_r dr$.
- Field lines cross an equipotential at right angles.

Your turn: hold an oil drop

- $m = 6.52 \times 10^{-15}$ kg; field 2.00×10^5 N/C downwards.
- The drop is stationary; neglect other forces.
- Find signed charge and the number of excess or missing electrons.

Separate magnitude from charge sign

1 Force balance

$$|q| = mg/E = [(6.52 \times 10^{-15} \text{ kg})(9.81 \text{ m/s}^2)]/(2.00 \times 10^5 \text{ N/C}) = 3.20 \times 10^{-19} \text{ C}$$

2 Sign and count $q < 0$;

$$n = |q|/e = (3.20 \times 10^{-19} \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 2$$

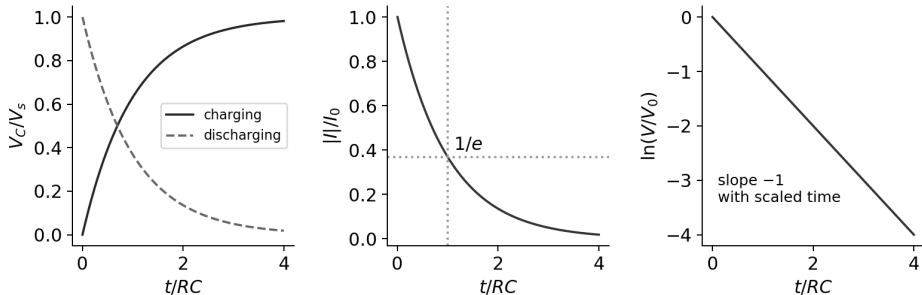
A dust-lift threshold needs volume and direction

- $m = \rho(4\pi r^3/3)$; vertical threshold $|q|E = mg$.
- A tilted panel needs $|q|E = mg \cos \theta$ for a field normal to it.
- Convert radius to diameter; choose the largest listed size below the threshold.

Capacitance and the half in energy

- Capacitance: $C = Q/V$, with Q on either plate.
- Energy: $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/(2C)$.
- Between two voltages: $\Delta W = \frac{1}{2}C(V_1^2 - V_2^2)$.

Different quantities need different RC curves



- Discharge voltage decays exponentially towards zero.
- A plot of $\ln(V/V_0)$ against time has gradient $-1/(RC)$.

Your turn: discharge and lost energy

- $C = 100 \mu\text{F}$, $R = 20.0 \text{ k}\Omega$, $V_0 = 12.0 \text{ V}$.
- Find voltage after 3.0 s and stored energy lost.
- Convert prefixes; retain the unrounded voltage for energy.

Exponential voltage; difference of energies

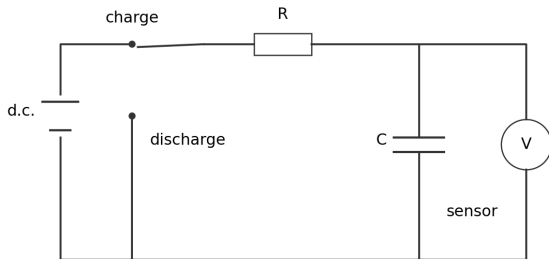
1 **Time constant** $\tau = RC = (20.0 \times 10^3 \, \Omega)(100 \times 10^{-6} \, \text{F}) = 2.00 \, \text{s}$

2 **Voltage** $V = V_0 e^{-t/RC} = 12.0 \, \text{V} \times e^{-3.0/2.00} = 2.68 \, \text{V}$

3 **Energy lost**

$$\Delta W = \frac{1}{2} C (V_0^2 - V^2) = \frac{1}{2} (100 \times 10^{-6} \, \text{F}) (12.0^2 - 2.68^2) \, \text{V}^2 = 6.84 \, \text{mJ}$$

CP11: measure the capacitor, not the supply



- Connect the voltmeter across the capacitor.
- Isolate the supply before timing discharge through the resistor.

Use a log fit to test resistance

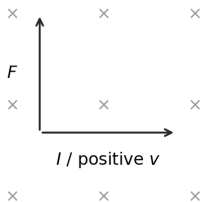
- $\ln(V/V_0) = -t/(RC)$; gradient $= -1/(RC)$.
- $R = -\Delta t/[C \ln(V_2/V_1)]$ for one discharge.
- Compare both tolerance limits; sensor loading changes the effective resistance.

RC extensions retain the same charge model

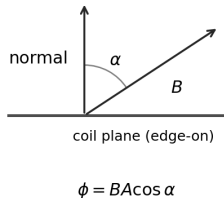
- Smaller load resistance: smaller RC , faster discharge between supply peaks, poorer smoothing.
- Identical series capacitors have equal charge magnitude and share the supply p.d. equally.
- For CP11, repeat from the same initial charge; avoid log fits near zero voltage.

Force angle and flux angle are different

Field into page; force upwards



Flux uses angle to area normal



- Magnetic flux density B : force uses the angle between velocity and field.
- Magnetic flux ϕ : use the angle between field and surface normal.
- Fleming's left-hand rule gives force for conventional current.

Flux linkage counts every linked turn

1 Coil area $A = \pi r^2 = \pi(0.015 \text{ m})^2 = 7.07 \times 10^{-4} \text{ m}^2$

2 Flux linkage $N\phi = NBA = 40(0.020 \text{ T})(7.07 \times 10^{-4} \text{ m}^2) = 5.65 \times 10^{-4} \text{ Wb}$

A balance reads the reaction on the magnets

- A separately supported wire experiences $F = BIl \sin \theta$.
- Magnets on the balance experience an equal, opposite reaction.
- Convert the balance change to force: $|F| = |\Delta m|g$.

Your turn: a changing flux

- 50 turns; flux per turn falls from $6.0 \times 10^{-5} \text{ Wb}$ to $2.0 \times 10^{-5} \text{ Wb}$.
- The change takes 0.020 s.
- Find average e.m.f. magnitude and describe the opposing magnetic effect.

Faraday magnitude; Lenz direction

- 1 Faraday's law** $|\mathcal{E}| = N|\Delta\phi|/\Delta t = 50|2.0 - 6.0| \times 10^{-5} \text{ Wb}/(0.020 \text{ s}) = 0.10 \text{ V}$
- 2 Lenz's law** Linked flux decreases; induced current, if a closed circuit exists, helps maintain it.

Moving boundaries turn speed into a flux rate

- Uniform field; boundary of width l moving at v : $\Delta A/\Delta t = lv$.
- Hence $|\mathcal{E}| = NB\Delta A/\Delta t = NBlv$.
- Reversing a magnetic strip's pole orientation reverses the signal sign.

Wireless charging needs changing linkage

- Electromagnetic induction: alternating current changes magnetic field.
- Changing receiver flux linkage $\rightarrow$ induced e.m.f.
- Some flux misses the receiver; equal turns need not give equal voltages.

Scattering links observation to a model

Observation / symbol	Supported conclusion
Most pass nearly straight	Most atomic volume is empty
Rare large deflections	Tiny, dense, positive nucleus
A_ZX	Proton number Z ; nucleon number A ; neutrons $A - Z$

Why a distributed positive charge was inadequate

- A distributed positive-charge model predicts predominantly small deflections.
- Rare large-angle alpha scattering requires a concentrated, strongly repulsive positive region.
- Few close encounters support a very small nucleus containing most atomic mass.

Thermionic emission supplies electrons

- Heating gives some electrons enough energy to leave metal.
- Electric acceleration: $\Delta E_k = q(V_i - V_f)$.
- From rest at low speed: $\frac{1}{2}mv^2 = |q||\Delta V|$.

From wavelength to acceleration voltage

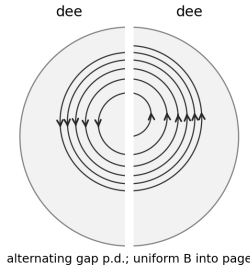
- For an electron at low speed, $p = h/\lambda$.
- Use $E_k = p^2/(2m_e)$, then accelerating p.d. magnitude $V = E_k/e$.
- Starting from rest is required for eV to equal the final kinetic energy.

Linear accelerator: timing determines tube length

- Electric field accelerates particles across gaps.
- Conducting drift tubes shield particles while polarity reverses.
- At fixed frequency, increasing speed needs longer tubes.

Cyclotron: electric acceleration, magnetic steering

Cyclotron: radius grows after each gap



- The gap electric field increases kinetic energy.
- The magnetic field bends motion without doing work.

Your turn: radius and half-turn time

- Proton: $p = 2.0 \times 10^{-20} \text{ kg m/s}$; $B = 0.40 \text{ T}$.
- $m_p = 1.67 \times 10^{-27} \text{ kg}$; $e = 1.60 \times 10^{-19} \text{ C}$.
- Find radius and time in one dee; distinguish half period from full period.

Radius grows; the classical period does not

1 Radius

$$r = p/(Be) = (2.0 \times 10^{-20} \text{ kg m/s}) / [(0.40 \text{ T})(1.60 \times 10^{-19} \text{ C})] = 0.313 \text{ m}$$

2 Period $T = 2\pi r/v = 2\pi m/(Be)$

3 One dee

$$t_{\text{dee}} = \pi m/(Be) = \pi(1.67 \times 10^{-27} \text{ kg}) / [(0.40 \text{ T})(1.60 \times 10^{-19} \text{ C})] = 8.20 \times 10^{-8} \text{ s}$$

Tracks provide evidence with limits

- Charged particles ionise matter; neutral products may carry missing momentum.
- $p = B|q|r$ for perpendicular tracks; sign needs field and travel direction.
- High energy gives short wavelength and allows new particles; fast muons have increased Earth-frame lifetime.

Curvature does not supply every missing direction

- Charge sign needs both magnetic-field direction and particle travel direction.
- A tightening track often indicates falling momentum; a constant-radius arc alone gives no travel arrow.
- Fast muons have longer mean lifetimes in the Earth frame; their speed remains below c .

The quark–lepton model organises particles

Family	Constituents / example
Baryon	Three quarks; proton uud
Meson	Quark + antiquark; pion
Lepton / antilepton	Electron / positron
Photon	Electromagnetic quantum

Quark charges make testable predictions

- Up-type charge $+2e/3$; down-type charge $-e/3$; antiquarks reverse signs.
- Proton uud : $+2e/3 + 2e/3 - e/3 = +e$; neutron udd : 0.
- Symmetry of quark families predicted the top quark before its observation.

Antiparticle: more than opposite charge

- Same rest mass; opposite charge when charged.
- Other additive quantum numbers also reverse.
- Neutral particle–antiparticle distinctions can remain.

Your turn: repair a decay equation

- Candidate: $n \rightarrow p + e^- + \nu_e$.
- Compare charge, baryon and lepton totals.
- If one total fails, choose the corrected neutrino symbol.

The bar changes lepton number

Total	n	$p + e^- + \bar{\nu}_e$
Charge / e	0	$+1-1+0 = 0$
Baryon	1	$1+0+0 = 1$
Lepton	0	$0+1-1 = 0$

Pair production and annihilation

- Mass–energy equivalence: $\Delta E = c^2 \Delta m$.
- Pair production supplies both rest masses, plus any kinetic/recoil energy.
- Annihilation at rest: two opposite equal-energy photons conserve momentum.

MeV/c squared is a mass unit

- 1 **Rest energy** $mc^2 = 140 \times 10^6 \times 1.60 \times 10^{-19} \text{ J} = 2.24 \times 10^{-11} \text{ J}$
- 2 **Mass** $m = E/c^2 = (2.24 \times 10^{-11} \text{ J}) / (3.00 \times 10^8 \text{ m/s})^2 = 2.49 \times 10^{-28} \text{ kg}$
- 3 **Compare** % difference = $100|m_{\text{predicted}} - m_{\text{reference}}|/m_{\text{reference}}$

Check yourself

- At the top of a vertical circle, which actual forces supply the inward resultant?
- During RC discharge, what does a log-voltage gradient measure?
- Can a magnetic field alone increase a particle's kinetic energy? Explain.
- A proposed decay conserves charge: what else must you test?