

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Compare the same line with its rest wavelength.

$$z = \frac{\lambda_o - \lambda_r}{\lambda_r} = \frac{(606 - 600) \text{ nm}}{600 \text{ nm}} = 0.010$$

Redshift.

$$v \approx zc_0 = 0.010(3.00 \times 10^8 \text{ m s}^{-1}) = 3.00 \times 10^6 \text{ m s}^{-1}$$

Speed. Observed frequency decreases.

2.2 Convert km to m and Mpc to m.

$$H_0 = \frac{65 \times 10^3 \text{ m s}^{-1}}{3.1 \times 10^{22} \text{ m}} = 2.10 \times 10^{-18} \text{ s}^{-1}$$

Rate.

$$t_H = \frac{1}{H_0} = \frac{1}{2.0968 \times 10^{-18} \text{ s}^{-1}} = 4.77 \times 10^{17} \text{ s}$$

Seconds.

$$t_{H, \text{years}} = \frac{t_H}{3.15 \times 10^7 \text{ s/year}} = \frac{4.7692 \times 10^{17} \text{ s}}{3.15 \times 10^7 \text{ s/year}} = 1.51 \times 10^{10} \text{ years}$$

Years.

2.3 Motion changes spacing/arrival interval of successive wavefronts along the line of sight. Recession stretches wavelength and reduces frequency; approach does the reverse. Identify the same atomic transition before comparing observed and rest wavelengths; different lines already have different rest wavelengths.

3.1 Rest wavelength is the midpoint 500.00 nm under the given symmetry. Each limb shift is half the full separation.

$$\Delta\lambda = \frac{\lambda_+ - \lambda_-}{2} = \frac{(500.02 - 499.98) \text{ nm}}{2} = 0.020 \text{ nm}$$

$$v \approx \frac{\Delta\lambda}{\lambda_0} c_0 = \frac{0.020 \text{ nm}}{500.00 \text{ nm}} (3.00 \times 10^8 \text{ m s}^{-1}) = 1.20 \times 10^4 \text{ m s}^{-1}$$

Speed. Use circumference per period.

$$T = \frac{2\pi R}{v} = \frac{2\pi(5.0 \times 10^8 \text{ m})}{1.20 \times 10^4 \text{ m s}^{-1}} = 2.62 \times 10^5 \text{ s}$$

Period. Full separation includes one red and one blue shift, so it doubles either limb's speed.

3.2 Use the small-redshift approximation.

$$v \approx zc_0 = 0.020(3.00 \times 10^8 \text{ m s}^{-1}) = 6.00 \times 10^6 \text{ m s}^{-1}$$

$$d = \frac{v}{H_0} = \frac{6.00 \times 10^6 \text{ m s}^{-1}}{2.2 \times 10^{-18} \text{ s}^{-1}} = 2.73 \times 10^{24} \text{ m}$$

Distance method/value. Nearby peculiar motion can dominate recession, and high redshift needs a fuller cosmological relation. Reciprocal present expansion rate is an age scale; actual age depends on expansion history rather than assuming constant-rate extrapolation.

3.3 Infer gravitational mass needed for observed orbital/rotation speeds. Motions can require more mass than luminous material accounts for, supporting a non-luminous gravitating component called dark matter. Dark energy describes the component/effect associated with accelerated cosmic expansion, not unseen gravitating mass alone. Future expansion depends on cosmic matter/energy content and how it evolves; current expansion plus uncertain content does not by itself establish certain recollapse or one exact future history.