

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Use Wien's wavelength law, converting nm to m.

$$T = \frac{2.898 \times 10^{-3} \text{ m K}}{\lambda_{max}} = \frac{2.898 \times 10^{-3} \text{ m K}}{500 \times 10^{-9} \text{ m}} = 5796 \text{ K}$$

Temperature. Higher T moves the wavelength peak shorter and increases total emitted power per area as T^4 for a black body.

2.2 Parallax is half the full six-month shift, 0.050 arcsecond.

$$d_{pc} = \frac{1}{p_{arcsec}} = \frac{1}{0.050} = 20 \text{ pc}$$

Distance. The full baseline is two Earth-orbit radii, while parallax uses one radius.

2.3 Rearrange the inverse-square intensity relation.

$$d = \sqrt{\frac{L}{4\pi I}} = \sqrt{\frac{8.0 \times 10^{28} \text{ W}}{4\pi(4.0 \times 10^{-12} \text{ W m}^{-2})}} = 3.99 \times 10^{19} \text{ m}$$

Substitution and value. Absorption by dust or anisotropic emission can invalidate the assumed relation.

3.1 Use the emitting surface for luminosity.

$$L = 4\pi R^2 \sigma T^4 = 4\pi(6.0 \times 10^8 \text{ m})^2(5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})(6000 \text{ K})^4 = 3.32 \times 10^{26} \text{ W}$$

Luminosity and unit. Spread that power over a sphere at observer distance.

$$I = \frac{L}{4\pi d^2} = \frac{3.324 \times 10^{26} \text{ W}}{4\pi(2.0 \times 10^{11} \text{ m})^2} = 661 \text{ W m}^{-2}$$

Intensity. R is stellar radius; d is star-to-planet distance, describing different physical spheres.

3.2 Sun near 5800 K, one solar luminosity, on main sequence. Red giant upper right, cool but luminous; white dwarf lower left, hot but faint. Large giant radius/surface area can more than offset lower T^4 in $L = 4\pi R^2 \sigma T^4$. A track records changing temperature and luminosity, not physical travel across space.

3.3 Gravity contracts a gas/dust cloud and heats its core. Hydrogen fusion sustains main sequence, with pressure balancing gravity. Depletion of core hydrogen causes core contraction and heating. Shell hydrogen burning and later helium fusion accompany red-giant expansion. Outer layers are lost, leaving a white dwarf without sustained fusion, which cools. A very massive star can fuse heavier nuclei before core collapse/supernova, leaving a neutron star or black hole rather than the Sun's white-dwarf outcome. Its hotter core consumes fuel much faster, so greater fuel supply need not outweigh its much greater fusion rate.