

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Divide Newtonian force by test mass.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(8.0 \times 10^{23} \text{ kg})}{(2.0 \times 10^6 \text{ m})^2} = 13.3 \text{ N kg}^{-1}$$

Value; radially inward.

2.2 Potential uses zero at infinity.

$$M = -\frac{Vr}{G} = -\frac{(-1.2 \times 10^7 \text{ J kg}^{-1})(3.0 \times 10^7 \text{ m})}{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}} = 5.40 \times 10^{24} \text{ kg}$$

Mass. Positive work is needed to take a unit mass to infinity; its finite-radius potential is below the chosen zero.

2.3 Both force magnitudes vary as inverse square outside a point/spherical source. Gravity acts on mass; electric interaction on charge. Gravity attracts, whereas charges may attract or repel and electric field direction is for a positive test charge. Both field strengths are vectors and both potentials scalars.

3.1 Use final minus initial radial potential energy.

$$\Delta E = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$
$$\Delta E = (3.6 \times 10^{14} \text{ m}^3 \text{ s}^{-2})(500 \text{ kg}) \left(\frac{1}{6.0 \times 10^6 \text{ m}} - \frac{1}{1.8 \times 10^7 \text{ m}} \right) = +2.00 \times 10^{10} \text{ J}$$

Magnitude, positive increase. Field varies substantially over this radial interval. Launch can also change kinetic energy and involve losses.

3.2 Set gravity equal to inward resultant.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad v = \frac{2\pi r}{T}$$

Eliminate v and cancel orbiting mass.

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad M = \frac{4\pi^2 r^3}{GT^2}$$

Derivation.

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (2.4 \times 10^7 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(4.0 \times 10^4 \text{ s})^2} = 5.11 \times 10^{24} \text{ kg}$$

Substitution and result.

3.3 Match rotation period and solve the orbit equation.

$$r = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} = \left[\frac{(8.0 \times 10^{13} \text{ m}^3 \text{ s}^{-2})(5.0 \times 10^4 \text{ s})^2}{4\pi^2} \right]^{1/3} = 1.72 \times 10^7 \text{ m}$$

Radius.

$$h = r - R = (1.7175 \times 10^7 - 3.0 \times 10^6) \text{ m} = 1.42 \times 10^7 \text{ m}$$

Height. Orbit must be circular, equatorial and in the planet's rotation direction with matching angular velocity. An inclined orbit changes latitude relative to the surface, so does not remain over one point.