

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 The steady forced frequency is 2.0 Hz. Forced oscillation is maintained by continued periodic driving; free oscillation follows a disturbance without continued periodic forcing.

2.2 Subtract transferred energy from the initial store.

$$E_2 = E_0 - 2\Delta E = (0.10 - 2 \times 0.020) \text{ J} = 0.060 \text{ J}$$

Energy. Spring energy is proportional to amplitude squared.

$$\frac{A_2}{A_0} = \sqrt{\frac{E_2}{E_0}} = \sqrt{\frac{0.060 \text{ J}}{0.10 \text{ J}}} = 0.775$$

Ratio. Constant loss per cycle is stated for these two cycles, not a universal damping law.

2.3 Ideal periods are equal because $T = 2\pi\sqrt{l/g}$ does not depend on mass. Equal length and angle give equal vertical rise; $E = mgh$ therefore gives energy ratio 1:3. Damping time also depends on the resistive mechanism and losses per cycle; those have not been specified.

3.1 Walking provides a periodic driving force. Near a natural frequency, energy transfer into the relevant bridge mode is efficient; its amplitude grows. Large amplitude is the danger, not a claim that the bridge frequency grows indefinitely. Dampers do work against motion, transferring mechanical energy to thermal stores and limiting steady amplitude.

3.2 Plastic deformation persists after unloading and transfers mechanical energy irreversibly into internal energy. An ideal elastic strip returns its stored energy, so storage alone is not loss/damping. Ductile material can undergo substantial plastic deformation before fracture; brittle material fractures with little plastic deformation.

3.3 Keep driving-force amplitude and oscillator mass/spring/support unchanged. Sweep driving frequency and measure settled amplitude at each frequency, sampling densely near the peak and repeating. Added damping lowers the peak and broadens it. The amplitude maximum depends on damping as well as restoring/inertial properties, so stronger damping can shift it below the undamped natural frequency; only light damping makes them approximately equal.