

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Use period to obtain angular frequency.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = 7.85 \text{ rad s}^{-1}$$

Angular frequency.

$$v_{\max} = A\omega = (0.040 \text{ m})(7.854 \text{ s}^{-1}) = 0.314 \text{ m s}^{-1}$$

Speed.

$$|a|_{\max} = A\omega^2 = (0.040 \text{ m})(7.854 \text{ s}^{-1})^2 = 2.47 \text{ m s}^{-2}$$

Acceleration.

2.2 Same spring gives squared period proportional to total mass.

$$m_2 = m_1(T_2/T_1)^2 = (0.20 \text{ kg})(0.90 \text{ s}/0.60 \text{ s})^2 = 0.45 \text{ kg}$$

Total mass.

$$m_{\text{added}} = m_2 - m_1 = (0.45 - 0.20) \text{ kg} = 0.25 \text{ kg}$$

Added mass.

2.3 At $T/4$, $x=0$, velocity points in the negative x direction, acceleration is zero. At $T/2$, $x=-A$ and $v=0$. Acceleration is always $-\omega^2 x$, towards equilibrium; the initial restoring acceleration produces the negative velocity.

3.1 The gradient is $-\omega^2$, so $\omega = 5.0 \text{ rad s}^{-1}$.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.0 \text{ s}^{-1}} = 1.26 \text{ s}$$

Period.

$$E_{k,\max} = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(0.30 \text{ kg})(25 \text{ s}^{-2})(0.060 \text{ m})^2 = 0.0135 \text{ J}$$

Energy method/value.

$$E_k = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}(0.30 \text{ kg})(25 \text{ s}^{-2})(0.060^2 - 0.020^2) \text{ m}^2 = 0.0120 \text{ J}$$

Displaced energy.

3.2 Use the small-angle pendulum model.

$$T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{0.80 \text{ m}}{9.81 \text{ m s}^{-2}}} = 1.79 \text{ s}$$

Period. Doubling mass does not change the ideal period. Time many complete oscillations and divide by their number; repeat at small amplitude, measuring pivot-to-centre length.

3.3 Transform frequencies to reciprocal squares and interpolate that variable.

$$y_1 = f_1^{-2} = (4.0 \text{ Hz})^{-2} = 0.06250 \text{ s}^2$$

$$y_2 = f_2^{-2} = (2.8 \text{ Hz})^{-2} = 0.12755 \text{ s}^2$$

$$y_u = f_u^{-2} = (3.2 \text{ Hz})^{-2} = 0.097656 \text{ s}^2$$

Transformed values.

$$m_u = m_1 + (m_2 - m_1) \frac{y_u - y_1}{y_2 - y_1}$$

$$m_u = 0.10 \text{ kg} + (0.30 - 0.10) \text{ kg} \frac{(0.097656 - 0.06250) \text{ s}^2}{(0.127551 - 0.06250) \text{ s}^2} = 0.208 \text{ kg}$$

Interpolation/value. Hanger/effective spring mass contributes a nonzero intercept. Keep the same spring and support with small amplitudes below elastic limit; vary frequency slowly around each steady peak and repeat the reading. Two points are provided for this task; a real calibration needs more to test linearity.