

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Fixed N and V give p/T constant. Convert temperatures to 300 K and 360 K.

$$p_2 = p_1 \frac{T_2}{T_1} = 1.2 \times 10^5 \text{ Pa} \frac{360 \text{ K}}{300 \text{ K}} = 1.44 \times 10^5 \text{ Pa}$$

Pressure. Gas-law proportionality uses absolute temperature; Celsius has an offset zero.

2.2 Use the molecule form of the ideal-gas equation.

$$N = \frac{pV}{k_B T} = \frac{(2.0 \times 10^5 \text{ Pa})(3.0 \times 10^{-3} \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})} = 1.45 \times 10^{23}$$

Molecule count; N counts individual particles, whereas amount in moles is N/N_A .

2.3 From $\frac{1}{2}m_0 c_{rms}^2 = \frac{3}{2}k_B T$, RMS speed is inversely proportional to square root of molecule mass at equal T .

$$\frac{c_l}{c_h} = \sqrt{\frac{m_h}{m_l}} = \sqrt{\frac{9m}{m}} = 3$$

Ratio 3:1. Mean translational kinetic energies are equal.

3.1 Radius is half diameter; first calculate volume.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.10 \text{ m})^3 = 4.19 \times 10^{-3} \text{ m}^3$$

$$N = \frac{pV}{k_B T} = \frac{(1.5 \times 10^5 \text{ Pa})(4.1888 \times 10^{-3} \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J K}^{-1})(290 \text{ K})} = 1.57 \times 10^{23}$$

Count.

$$m_g = Nm_0 = (1.5700 \times 10^{23})(4.65 \times 10^{-26} \text{ kg}) = 7.30 \times 10^{-3} \text{ kg}$$

Gas mass. Use mean molecular kinetic energy to find RMS speed.

$$c_{rms} = \sqrt{\frac{3k_B T}{m_0}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J K}^{-1})(290 \text{ K})}{4.65 \times 10^{-26} \text{ kg}}} = 508 \text{ m s}^{-1}$$

Speed and unit.

3.2 Elastic collision gives momentum transfer $2m_0 c_x$. The return interval to the same wall is $2l/c_x$; mean force is $m_0 c_x^2/l$. Sum N molecules, divide by wall area and use equal squared components in 3 directions:

$$pV = Nm_0 \langle c_x^2 \rangle = \frac{1}{3}Nm_0 \langle c^2 \rangle$$

Equating with $Nk_B T$ gives $\frac{1}{2}m_0 \langle c^2 \rangle = \frac{3}{2}k_B T$. Negligible particle volume; negligible forces between particles except elastic collisions.

3.3 Use absolute pressure and total connected volume.

$$p_{1,abs} = p_{1,g} + p_{atm} = (80 + 100) \text{ kPa} = 180 \text{ kPa}$$

$$p_{2,abs} = p_{1,abs} \frac{V_{1,total}}{V_{2,total}} = 180 \text{ kPa} \frac{(20 + 5) \text{ cm}^3}{(35 + 5) \text{ cm}^3} = 112.5 \text{ kPa}$$

Absolute result.

$$p_{2,g} = p_{2,abs} - p_{atm} = (112.5 - 100) \text{ kPa} = 12.5 \text{ kPa}$$

Gauge result. Keep the same sealed amount of gas; change volume slowly and wait for thermal equilibration. If final gas remains warmer, pressure is higher than this isothermal prediction.