

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 The known mass and temperature rise require sensible heating.

$$\Delta E = mc\Delta\theta = (0.25 \text{ kg})(900 \text{ J kg}^{-1} \text{ K}^{-1})(50 - 20) \text{ K} = 6750 \text{ J}$$

Energy. Temperature relates to average random kinetic energy; total internal energy also depends on amount and intermolecular potential energy/state.

2.2 At constant temperature, use phase-change energy.

$$L = \frac{Pt}{\Delta m} = \frac{(24 \text{ W})(300 \text{ s})}{0.020 \text{ kg}} = 3.6 \times 10^5 \text{ J kg}^{-1}$$

Value. If some heater energy escapes to surroundings, less causes melting; assigning all input to melting overestimates L. This assumes net loss, not extra melting caused by warm surroundings.

2.3 Output is across the lower resistor, so use the divider relation.

$$V_{out,c} = V_s \frac{R_c}{R + R_c} = 6.0 \text{ V} \frac{4.0 \text{ k}\Omega}{(2.0 + 4.0) \text{ k}\Omega} = 4.0 \text{ V}$$

Cold value.

$$V_{out,w} = V_s \frac{R_w}{R + R_w} = 6.0 \text{ V} \frac{1.0 \text{ k}\Omega}{(2.0 + 1.0) \text{ k}\Omega} = 2.0 \text{ V}$$

Warm value. Warming reduces lower resistance and its share of supply voltage.

3.1 Melting and warming the resulting water are separate stores.

$$E_u = mL_f + mc_w\Delta\theta$$

$$E_u = (0.15 \text{ kg})(3.3 \times 10^5 \text{ J kg}^{-1}) + (0.15 \text{ kg})(4200 \text{ J kg}^{-1} \text{ K}^{-1})(20 \text{ K}) = 62100 \text{ J}$$

Useful energy.

$$E_{in} = Pt = (200 \text{ W})(400 \text{ s}) = 80000 \text{ J}$$

Input energy.

$$\eta = \frac{E_u}{E_{in}} = \frac{62100 \text{ J}}{80000 \text{ J}} = 0.776 = 77.6\%$$

Efficiency.

$$P_{loss} = \frac{E_{in} - E_u}{t} = \frac{(80000 - 62100) \text{ J}}{400 \text{ s}} = 44.8 \text{ W}$$

Loss power. Energy during melting changes molecular arrangement/potential energy, without raising average random kinetic energy/temperature during the ideal phase change.

3.2 At either steady power, $P = L\dot{m} + P_{loss}$. Subtract to eliminate the same loss power.

$$L = \frac{P_2 - P_1}{\dot{m}_2 - \dot{m}_1} = \frac{(60 - 40) \text{ W}}{(2.1 - 1.2) \times 10^{-5} \text{ kg s}^{-1}} = 2.22 \times 10^6 \text{ J kg}^{-1}$$

Numerical value and unit. Heat-loss power must be approximately unchanged at the two steady boiling conditions.

3.3 Place an independent thermometer beside the thermistor in a stirred water bath. Record multiple temperatures and outputs, waiting for equilibrium each time. Keep supply and fixed resistance unchanged. Plot a calibration and interpolate output at 45 °C; the NTC relation may curve, so end points alone do not establish the switching voltage. Use a high-input-resistance voltmeter to avoid loading. Keep contacts dry/insulated and use low voltage with hot-water precautions.