

Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Proton: baryon; pion: meson. Electron: lepton; positron: antilepton. Photon: neither baryon nor lepton. Proton is uud , whose charges sum $2/3 + 2/3 - 1/3 = +1$ in units of e .

2.2 Equal rest mass and opposite electric charge. A neutral antiparticle can have opposite additive quantum numbers, for example lepton number, despite both charges being zero. The pattern/symmetry of the established quark families suggested a missing partner with predicted properties, later tested experimentally as the top quark.

2.3 Convert MeV to joules then divide by c^2 .

$$m = \frac{E}{c^2} = \frac{135 \times 10^6 \times 1.60 \times 10^{-19} \text{ J}}{(3.00 \times 10^8 \text{ m s}^{-1})^2} = 2.40 \times 10^{-28} \text{ kg}$$

Mass. Compare to the specified reference value.

$$\% \text{ difference} = 100 \frac{|m_{\text{pred}} - m_{\text{ref}}|}{m_{\text{ref}}} = 100 \frac{|2.35 - 2.40| \times 10^{-28} \text{ kg}}{2.40 \times 10^{-28} \text{ kg}} = 2.08\%$$

Percentage difference.

3.1 Both equations conserve charge: $0 = +1 - 1 + 0$. Both conserve baryon number: $1 = 1 + 0 + 0$. With neutrino, final lepton number is $0 + 1 + 1 = 2$, unlike initial zero. With antineutrino, final total is $0 + 1 - 1 = 0$, so the second passes all three checks. These are necessary constraints; passing them alone does not establish the interaction mechanism or probability.

3.2 Both rest masses contribute.

$$E_{\text{total}} = 2m_e c^2 = 2 \times 9.11 \times 10^{-31} \text{ kg} \times (3.00 \times 10^8 \text{ m s}^{-1})^2 = 1.64 \times 10^{-13} \text{ J}$$

Total energy.

$$E_\gamma = \frac{E_{\text{total}}}{2} = \frac{1.6398 \times 10^{-13} \text{ J}}{2} = 8.20 \times 10^{-14} \text{ J}$$

Each energy. Photon momenta must be opposite and equal in magnitude for initial zero momentum, so energies are equal. One photon with nonzero energy has nonzero momentum and cannot alone balance initial zero momentum.

3.3 Both rest masses are required.

$$E_{\text{min}} = 2m_e c^2 = 2 \times 9.11 \times 10^{-31} \text{ kg} \times (3.00 \times 10^8 \text{ m s}^{-1})^2 = 1.64 \times 10^{-13} \text{ J}$$

$$E_{\text{min}} = \frac{2m_e c^2}{1.60 \times 10^{-13} \text{ J per MeV}} = \frac{1.6398 \times 10^{-13} \text{ J}}{1.60 \times 10^{-13} \text{ J per MeV}} = 1.025 \text{ MeV}$$

Threshold approximately 1.02 MeV.

$$E_{k,\text{max}} = E_\gamma - 2m_e c^2 = (1.20 - 1.024875) \text{ MeV} = 0.175 \text{ MeV}$$

Available kinetic energy. The nucleus takes recoil momentum, allowing total momentum conservation; finite recoil reduces available pair kinetic energy. A photon has zero rest mass and travels at c ; use photon momentum E/c , not the massive non-relativistic formula mv . Pair baryon total stays zero; electron/positron lepton numbers $+1, -1$ sum to zero as before.