

Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Protons 79, neutrons $197 - 79 = 118$, electrons 79 for the neutral atom. Most alpha particles pass nearly straight through: most atomic volume is empty. Rare very large deflections require strong repulsion concentrated in a tiny positive nucleus containing most mass, unlike a diffuse-positive-charge model.

2.2 Heating gives some electrons enough energy to leave a metal surface. Electric work becomes kinetic energy.

$$\frac{1}{2}m_e v^2 = eV \quad \Rightarrow \quad v = \sqrt{\frac{2eV}{m_e}}$$
$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \text{ C} \times 300 \text{ V}}{9.11 \times 10^{-31} \text{ kg}}} = 1.03 \times 10^7 \text{ m s}^{-1}$$

Rearrangement and speed. This is about $0.034c$, well below light speed, so the approximation is reasonable.

2.3 Magnetic force supplies centripetal resultant.

$$r = \frac{p}{eB} = \frac{3.0 \times 10^{-20} \text{ kg m s}^{-1}}{1.60 \times 10^{-19} \text{ C} \times 0.50 \text{ T}} = 0.375 \text{ m}$$

Radius. Half the non-relativistic period.

$$t_{1/2} = \frac{\pi m_p}{eB} = \frac{\pi \times 1.67 \times 10^{-27} \text{ kg}}{1.60 \times 10^{-19} \text{ C} \times 0.50 \text{ T}} = 6.56 \times 10^{-8} \text{ s}$$

Half-circle time.

3.1 Add the energy gains of all four gaps.

$$\frac{1}{2}m_p v^2 = 4eV_{\text{gap}} \quad \Rightarrow \quad v = \sqrt{\frac{8eV_{\text{gap}}}{m_p}}$$
$$v = \sqrt{\frac{8eV_{\text{gap}}}{m_p}} = \sqrt{\frac{8 \times 1.60 \times 10^{-19} \text{ C} \times 20 \times 10^3 \text{ V}}{1.67 \times 10^{-27} \text{ kg}}} = 3.92 \times 10^6 \text{ m s}^{-1}$$

Rearrangement and speed. At fixed frequency particles must spend approximately the same half-period inside each tube, so faster motion requires longer tubes. Polarity reverses while the particle is inside so the next gap again accelerates it. Conducting tubes shield the particle from the reversing electric field inside; energy changes mainly in gaps.

3.2 Electric field accelerates across the gap, increasing speed/energy. Perpendicular magnetic force curves motion within dees without doing work. Combining $qvB = mv^2/r$ with $T = 2\pi r/v$ gives:

$$T = \frac{2\pi m}{|q|B}$$

At fixed m, q, B , period is independent of speed; radius grows in proportion to speed and time per dee remains $T/2$. Alternating gap polarity reverses each half-turn to accelerate

again. Relativistic effects alter the required timing near c , so the fixed-frequency non-relativistic synchronisation fails.

3.3 Vector momentum conservation requires subtracting all visible outgoing momenta.

$$\mathbf{p}_{\text{miss}} = \mathbf{p}_{\text{in}} - \mathbf{p}_1 - \mathbf{p}_2 = (1.0, -0.50) \times 10^{-20} \text{ kg m s}^{-1}$$

Components.

$$p_{\text{miss}} = \sqrt{p_x^2 + p_y^2} = \sqrt{1.0^2 + 0.50^2} \times 10^{-20} \text{ kg m s}^{-1} = 1.12 \times 10^{-20} \text{ kg m s}^{-1}$$

Magnitude. Neutral particles do not directly ionise along a track in this detector model; later charged products may reveal them. High momentum gives shorter de Broglie wavelength and finer structural resolution. High collision energy can also create new massive particles. Fast muons have a longer average lifetime in the Earth's frame at speeds close to, but below, c .