

Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Convert to frequency.

$$f = \frac{N}{t} = \frac{90}{60 \text{ s}} = 1.50 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 1.50 \text{ s}^{-1} = 9.42 \text{ rad s}^{-1}$$

Angular speed. Use radius for linear speed.

$$v = \omega r = 3\pi \text{ s}^{-1} \times 0.40 \text{ m} = 3.77 \text{ m s}^{-1}$$

Tangential speed.

$$a = \omega^2 r = (3\pi \text{ s}^{-1})^2 \times 0.40 \text{ m} = 35.5 \text{ m s}^{-2}$$

Acceleration.

2.2 Angular speeds are equal. From $v = \omega r$, outer tangential speed is twice inner. From $a = \omega^2 r$, outer acceleration is twice inner. Velocity direction changes, so its vector changes despite constant magnitude.

2.3 The change is $\mathbf{v}_2 - \mathbf{v}_1$, drawn between the tips of vectors with a common tail. Similar small-angle triangles give $\Delta v/v \approx \Delta s/r$. With $\Delta s \approx v\Delta t$, divide by time and take the limit:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

The change is inward; addition instead describes a different vector and does not compare final with initial velocity.

3.1 Use the resultant of actual forces.

$$R_b = \frac{mv^2}{r} + mg = \frac{0.15 \text{ kg} \times (3.0 \text{ m s}^{-1})^2}{0.30 \text{ m}} + 0.15 \text{ kg} \times 9.81 \text{ m s}^{-2} = 5.97 \text{ N}$$

Bottom reaction.

$$R_t = \frac{mv^2}{r} - mg = \frac{0.15 \text{ kg} \times (3.0 \text{ m s}^{-1})^2}{0.30 \text{ m}} - 0.15 \text{ kg} \times 9.81 \text{ m s}^{-2} = 3.03 \text{ N}$$

Top equation and result. Weight opposes the required inward resultant at the bottom but contributes at the top, so reaction differs. Initial velocity is tangent; gravity can bend the later trajectory.

3.2 Friction provides the inward resultant; use its limiting value.

$$mr\omega^2 \leq \frac{mg}{20} \Rightarrow r_{\max} = \frac{g}{20\omega^2}$$

Rearranged limit.

$$r_{\max} = \frac{g}{20\omega^2} = \frac{9.81 \text{ m s}^{-2}}{20(2.0 \text{ s}^{-1})^2} = 0.123 \text{ m}$$

Calculation. Required and limiting forces both scale with mass, so mass cancels and doubling it leaves the radius unchanged.

3.3 Centripetal force is the resultant of real forces, not another force. At fixed mass/radius, $F = 4\pi^2 m r f^2$, so doubling frequency quadruples force. At the limiting top speed the reaction is zero.

$$mg = \frac{mv_{\min}^2}{r} \Rightarrow v_{\min} = \sqrt{gr}$$

$$v_{\min} = \sqrt{gr} = \sqrt{9.81 \text{ m s}^{-2} \times 0.50 \text{ m}} = 2.21 \text{ m s}^{-1}$$

Minimum speed.